

## Miscellaneous Ex 2 p 30-33

1a.  $5\sqrt{2} + 5 - 4\sqrt{2} + 2$   
 $= 7 + \sqrt{2}$

b.  $(2^{\frac{1}{2}})^4 + (3^{\frac{1}{2}})^4 + (4^{\frac{1}{2}})^4$   
 $= 2^2 + 3^2 + 4^2 = 4 + 9 + 16 = 29$

c.  $5 - 4\sqrt{5} + 4 + 5 - 4$   
 $= 10 - 4\sqrt{5}$

d.  $(2\sqrt{2})(2\sqrt{2})(2\sqrt{2})(2\sqrt{2})(2\sqrt{2})$   
 $2^5 (2^{\frac{1}{2}})^5 = 32 \times 2^{\frac{5}{2}} = 32 \times 2^2 \times 2^{\frac{1}{2}} = 128\sqrt{2}$

2a.  $3\sqrt{3} + 2\sqrt{3} - \sqrt{3} = 4\sqrt{3}$

b.  $3\sqrt{7} - 2\sqrt{7} = \sqrt{7}$

c.  $\sqrt{100 \times 100 \times 10} + \sqrt{10 \times 10 \times 10} + \sqrt{10}$   
 $= 100\sqrt{10} + 10\sqrt{10} + \sqrt{10} = 111\sqrt{10}$

d.  $\sqrt[3]{2} + \sqrt[3]{8 \times 2} = \sqrt[3]{2} + 2\sqrt[3]{2} = 3\sqrt[3]{2}$

3c.  $\frac{2\sqrt{5}}{3\sqrt{10}} = \frac{2}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{6} = \frac{1}{3}\sqrt{2}$

d.  $\frac{\sqrt{8} \times \sqrt{15}}{\sqrt{15} \times \sqrt{15}} = \frac{\sqrt{120}}{15} = \frac{2\sqrt{30}}{15}$

4c.  $\frac{1}{\sqrt{2}}(2\sqrt{2} - 1) + \sqrt{2}(1 - \sqrt{8})$   
 $= 2 - \frac{1}{\sqrt{2}} + \sqrt{2} - 4$   
 $= -2 - \frac{\sqrt{2}}{2} + \sqrt{2} \times \frac{2}{2}$   
 $= \frac{\sqrt{2}}{2} - 2$

d.  $\frac{\sqrt{6}}{\sqrt{2}} + \frac{3}{\sqrt{3}} + \frac{\sqrt{15}}{\sqrt{5}} + \frac{\sqrt{18}}{\sqrt{6}}$   
 $= \sqrt{3} + \sqrt{3} + \sqrt{3} + \sqrt{3}$   
 $= 4\sqrt{3}$

5.  $\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3}{5}\sqrt{5}$

6.  $\sqrt{12} \times \sqrt{75}$   
a.  $\sqrt{12 \times 75} = \sqrt{3 \times 4 \times 3 \times 25}$   
 $= 3 \times 10 = \underline{30}$

b.  $(3 \times 2^2)^{\frac{1}{2}} \times (3 \times 5^2)^{\frac{1}{2}}$   
 $= 3^{\frac{1}{2}} \times 2 \times 3^{\frac{1}{2}} \times 5 = 3 \times 2 \times 5$   
 $= \underline{30}$

8.a Area =  $\frac{1}{2}(6 - 2\sqrt{2})(6 + 2\sqrt{2})$   
 $= \frac{1}{2}(36 - 8) = \frac{1}{2} \times 28 = \underline{14 \text{ cm}^2}$

b.  $PR^2 = (6 - 2\sqrt{2})^2 + (6 + 2\sqrt{2})^2$   
 $= 36 - 24\sqrt{2} + 8 + 36 + 24\sqrt{2} + 8$   
 $= 72 + 16 =$   
 $PR = \sqrt{88} = 2\sqrt{22}$

9.  $\sqrt[3]{36} \times \sqrt[6]{\frac{4}{3}} \times \sqrt{27}$   
 $= (2^2 \times 3^2)^{\frac{1}{3}} \times (2^2 \times 3^{-1})^{\frac{1}{6}} \times (3^3)^{\frac{1}{2}}$   
 $= 2^{\frac{2}{3}} \times 3^{\frac{2}{3}} \times 2^{\frac{1}{3}} \times 3^{-\frac{1}{6}} \times 3^{\frac{3}{2}}$   
 $= 2 \times 3^{\frac{2}{3} - \frac{1}{6} + \frac{3}{2}}$   
 $= 2 \times 3^{\frac{4}{6} - \frac{1}{6} + \frac{9}{6}} = 2 \times 3^2 = \underline{18}$

10 AB =  $4\sqrt{3}$

BC =  $5\sqrt{3}$

$\angle B = 60^\circ$

Cosine rule -

$$AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cdot \cos \angle B$$

$$AC^2 = (4\sqrt{3})^2 + (5\sqrt{3})^2 - 2 \cdot 4\sqrt{3} \cdot 5\sqrt{3} \cdot \frac{1}{2}$$

$$= 48 + 75 - \frac{120}{2} = 123 - 60 = 63$$

$$AC = \sqrt{63} = \underline{3\sqrt{7}}$$

$$11 \quad 5x - 3y = 41 \quad \times 7\sqrt{2} \quad 35\sqrt{2}x - 21\sqrt{2}y = 287\sqrt{2}$$

$$7\sqrt{2}x + 4\sqrt{2}y = 82 \quad \times 5 \quad 35\sqrt{2}x + 20\sqrt{2}y = 410$$

$$4\sqrt{2}y = 410 - 287\sqrt{2}$$

$$y = \frac{410 - 287\sqrt{2}}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{410\sqrt{2}}{4 \cdot 2} - \frac{287 \cdot 2}{4 \cdot 2} = \frac{5\sqrt{2} - 7}{1}$$

$$5x - 3(5\sqrt{2} - 7) = 41$$

$$5x - 15\sqrt{2} + 21 = 41$$

$$5x = 20 + 15\sqrt{2}$$

$$x = 4 + 3\sqrt{2}$$

$$14 \text{ a } A(2,3) \quad m = -\frac{1}{2}$$

$$y - 3 = -\frac{1}{2}(x - 2)$$

$$2y - 6 = -x + 2$$

$$l \equiv x + 2y = 8 \Rightarrow m = -\frac{1}{2}$$

$$\text{b } P(2+2t, 3-t)$$

$$2+2t + 2(3-t) = 8$$

$$2+2t+6-2t = 8$$

$$8 = 8 \rightarrow \therefore P \text{ on } l \text{ whatever}$$

the value of  $t$

$$\text{c } AP = 5 \text{ units}$$

$$A(2,3) \quad P(2+2t, 3-t)$$

$$5^2 = (2+2t-2)^2 + (3-t-3)^2$$

$$25 = (2t)^2 + (-t)^2$$

$$25 = 4t^2 + t^2$$

$$25 = 5t^2$$

$$t^2 = 5$$

$$t = \pm \sqrt{5}$$

$$\text{d } OP \perp l$$

$$m_{OP} = 2 \quad (0,0)$$

$$P(2+2t, 3-t)$$

$$m_{OP} = \frac{3-t}{2+2t} = 2$$

$$3-t = 4+4t$$

$$3-4 = 5t$$

$$t = -\frac{1}{5}$$

$$x_p = 2+2t = 2+2\left(-\frac{1}{5}\right) = 2-\frac{2}{5} = \frac{8}{5}$$

$$y_p = 3-t = 3-\left(-\frac{1}{5}\right) = 3+\frac{1}{5} = \frac{16}{5}$$

$$\therefore P\left(\frac{8}{5}, \frac{16}{5}\right)$$

$$O-l = OP$$

$$= \sqrt{\left(\frac{8}{5}\right)^2 + \left(\frac{16}{5}\right)^2} = \sqrt{\frac{64}{25} + \frac{256}{25}}$$

$$= \sqrt{\frac{320}{25}} = \frac{8\sqrt{5}}{5}$$

15  $\frac{x}{a} + \frac{y}{b} = 1$   
 x-axis intercept:  $P(\dots, 0)$   
 $y = 0 \quad \frac{x}{a} + 0 = 1$   
 $\frac{x}{a} = 1$

$x = a, y = 0$   
 $P(a, 0)$

y-axis intercept:  $Q(0, \dots)$

$x = 0 \quad 0 + \frac{y}{b} = 1$   
 $y = b, x = 0$   
 $Q(0, b)$   
 $P(a, 0)$

$m_{PQ} = -3 = \frac{b}{-a}$

$b = 3a \quad (1)$

$PQ = 20$

$20^2 = \sqrt{a^2 + b^2} \quad (2)$

$400 = a^2 + (3a)^2$

$400 = a^2 + 9a^2 = 10a^2$

$a^2 = 40$

$a = 2\sqrt{10}$

$b = 3(2\sqrt{10}) = 6\sqrt{10} //$

17.a.  $2 + 4 = 6$

b.  $(2^5)^{-\frac{4}{5}} = 2^{-4} = \frac{1}{16}$

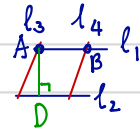
c.  $[(2^2)^{\frac{3}{2}}]^{-\frac{1}{3}}$

$= [2^3]^{-\frac{1}{3}} = 2^{-1} = \frac{1}{2}$

d.  $(\frac{16}{9})^{\frac{3}{2}} - (\frac{4}{3})^2]^{\frac{3}{2}} - (\frac{4}{3})^3 = \frac{64}{27}$

18.  $(\frac{27a^6}{b^3})^{-\frac{2}{3}} = [\frac{3^3 a^6}{b^3}]^{-\frac{2}{3}} = \frac{3^{-2} a^{-4}}{b^{-2}} = \frac{b^2}{9a^4}$

16  $l_1: y = 2x - 4$   
 $l_2: y = 2x - 13$   
 $l_3: x + y = 5$   
 $l_4: x + y = -4$



$l_1 \times l_3$

$2x - y = 4$

$x + y = 5$

$3x = 9$

$x = 3$

$y = 2 \quad (3, 2) \text{ A}$

$l_1 \times l_4$

$2x - y = 4$

$x + y = -4$

$3x = 0$

$x = 0$

$y = -4 \quad (0, -4) \text{ B}$

length  $= \sqrt{9 + 36} = \sqrt{45} = 3\sqrt{5}$

$A - l_2 \quad m = -\frac{1}{2}$

$y - 2 = -\frac{1}{2}(x - 3)$

$2y - 4 = -x + 3$

$x + 2y = 7 \Leftrightarrow 2x + 4y = 14$

$l_2: y = 2x - 13 \quad \frac{2x - y = 13}{5y = 1}$

$y = \frac{1}{5}$

$x + 2(\frac{1}{5}) = 7$

$x = 7 - \frac{2}{5} = \frac{33}{5} \quad D(\frac{33}{5}, \frac{1}{5})$

$A(3, 2)$

$AD = \sqrt{36^2 + 18^2} = \frac{9}{5}\sqrt{5}$

$A = \frac{9}{5}\sqrt{5} \times 3\sqrt{5} = 27 \text{ sq unit}$

19.  $(x^{\frac{1}{2}})^2 - 6x^{\frac{1}{2}} + 8 = 0$

$A^2 - 6A + 8 = 0$

$(A - 4)(A - 2) = 0$

$A = 4 \quad \text{or} \quad A = 2$

$x^{\frac{1}{2}} = 4 \quad x^{\frac{1}{2}} = 2$

$x = 4^2 = 16 \quad x = 2^2 = 4$

$$20 \quad 4^{2x} \times 8^{x-1} = 32$$

$$2^{4x} \times 2^{3(x-1)} = 2^5$$

$$2^{7x-3} = 2^5$$

$$7x-3=5$$

$$7x=8$$

$$x = \frac{8}{7}$$

$$22.a. \quad 4^{\frac{1}{2}} p^{\frac{1}{3}} q^{-\frac{2}{3}} = \frac{2p^{\frac{1}{3}}}{q^{\frac{2}{3}}}$$

$$b. \quad \frac{1}{5b \cdot 2 \cdot b^2} = \frac{1}{10b^3}$$

$$c. \quad 2^{\frac{1}{4}} x^{\frac{3}{2}} y^1 \times 2^{\frac{3}{4}} x^{-\frac{1}{2}}$$

$$= 2xy^2$$

$$d. \quad m^{\frac{2}{3}} n \times m^{\frac{2}{3}} n^{\frac{4}{3}} \times m^{-2} n^{-2}$$

$$= m^{\frac{4}{3}-2} \times n^{\frac{7}{3}-2} = m^{-\frac{2}{3}} n^{\frac{1}{3}} = \frac{n^{\frac{1}{3}}}{m^{\frac{2}{3}}}$$

$$23 \quad 3^{236} \approx 4 \times 10^{112}$$

$$3^{-376} \approx 4 \times 10^{-180}$$

$$a \quad 3^{376} \approx \frac{1}{4 \times 10^{-180}} = 0.25 \times 10^{180}$$

$$\approx 2.5 \times 10^{179}$$

$$b \quad 3^{612} = 3^{236} \times 3^{376}$$

$$\approx 4 \times 10^{112} \times 2.5 \times 10^{179}$$

$$\approx 10 \times 10^{112} \cdot 10^{179} = 10^{292}$$

$$c \quad (\sqrt{3})^{236} = 3^{\frac{236}{2}} \approx (4 \times 10^{112})^{\frac{1}{2}}$$

$$\approx 2 \times 10^{56}$$

$$d \quad (3^{-376})^{\frac{5}{2}} \approx (4 \times 10^{-180})^{\frac{5}{2}}$$

$$\approx 32 \times 10^{-450}$$

$$\approx 3.2 \times 10^{-449}$$

$$25 a \quad 2^{-\frac{3}{2}} + 2^{-\frac{1}{2}} + 2^{\frac{1}{2}} + 2^{\frac{3}{2}}$$

$$= \frac{1}{2\sqrt{2}} + \frac{1}{\sqrt{2}} + \sqrt{2} + 2\sqrt{2}$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{2} + \frac{4\sqrt{2}}{4} + \frac{8\sqrt{2}}{4} = \frac{15\sqrt{2}}{4}$$

$$b \quad (\sqrt{3})^{-3} + (\sqrt{3})^{-2} + (\sqrt{3})^{-1} + (\sqrt{3})^0 + (\sqrt{3})^1 + (\sqrt{3})^2 + (\sqrt{3})^3$$

$$= \frac{1}{3\sqrt{3}} + \frac{1}{3} + \frac{1}{\sqrt{3}} + 1 + \sqrt{3} + 3 + 3\sqrt{3}$$

$$= \frac{\sqrt{3}}{9} + \frac{1}{3} + \frac{3\sqrt{3}}{9} + 4 + 4\sqrt{3}$$

$$= \frac{4\sqrt{3}}{9} + \frac{1}{3} + 4 + 4\sqrt{3}$$

$$= \frac{13}{3} + \frac{40\sqrt{3}}{9}$$

$$a = \frac{13}{3} \quad b = \frac{40}{9}$$

$$26 a \quad 2^{2^{70}} = 2^{71}$$

$$b \quad 2 \cdot 2^{-400} = 2^{-399}$$

$$c \quad 4 \cdot 2^{\frac{1}{3}} = 2^2 \cdot 2^{\frac{1}{3}} = 2^{\frac{7}{3}}$$

$$d \quad 2^{100} - 2^{99} = 2 \cdot 2^{99} - 2^{99} = 2^{99}$$

$$e \quad 8 \times 8^{01} = 8^{11} = (2^3)^{11} = 2^{33}$$

$$27 \quad \frac{125^{3x}}{5^{x+4}} = \frac{25^{x-2}}{3125}$$

$$\frac{5^{9x}}{5^{x+4}} = \frac{5^{2(x-2)}}{5^5}$$

$$9x - (x+4) = 2x - 4 - 5$$

$$8x - 2x = 4 - 4 - 5$$

$$6x = -5$$

$$x = -\frac{5}{6}$$