Exercise 4A p 56

4. a. x^2+x+2
$= (x+1)^2 - 1 + 2$
$= (x+1)^2 + 1$

b. x^2-8x-3
$= (x-4)^2 - 16 - 3$
$= (x-4)^2 - 19$

c. x^2+3x-7
$= \left(x+\frac{3}{2}\right)^2 - \frac{9}{4} - 7$
$= \left(x+\frac{3}{2}\right)^2 - \frac{37}{4}$

d. $5-6x+x^2$
$= (x-3)^2 - 9 + 5$
$= (x-3)^2 - 4$

4e $x^2+14x+49$
$= (x+7)^2 - 49 + 49$
$= (x+7)^2$

f. $2x^2+12x-5$
$= 2(x^2+6x) - 5$
$= 2[(x+3)^2 - 9] - 5$
$= 2(x+3)^2 - 23$

g $3x^2-12x+3$
$= 3(x^2-4x) + 3$
$= 3[(x-2)^2 - 4] + 3$
$= 3(x-2)^2 - 12 + 3$
$= 3(x-2)^2 - 9$

h $7-8x-4x^2$
$= 7 - 4(x^2+2x)$
$= 7 - 4[(x+1)^2 - 1]$
$= 7 - 4(x+1)^2 + 4$
$= 11 - 4(x+1)^2$

i $2x^2+5x-3$
$= 2\left(x^2+\frac{5}{2}x\right) - 3$
$= 2\left[\left(x+\frac{5}{4}\right)^2 - \frac{25}{16}\right] - 3$
$= 2\left(x+\frac{5}{4}\right)^2 - \frac{25}{8} - \frac{24}{8}$
$= 2\left(x+\frac{5}{4}\right)^2 - \frac{49}{8}$
$=$

5b $x^2-14x-176$
$= (x-7)^2 - 49 - 176$
$= (x-7)^2 - 225$
$= (x-7+15)(x-7-15)$
$= (x+8)(x-22)$

d $6x^2-5x-6$
$= 6\left(x^2-\frac{5}{6}x\right) - 6$
$= 6\left[\left(x-\frac{5}{12}\right)^2 - \frac{25}{144}\right] - 6$
$= 6\left[\left(x-\frac{5}{12}\right)^2 - \frac{25}{144} - 1\right]$
$= 6\left[\left(x-\frac{5}{12}\right)^2 - \frac{169}{144}\right]$
$= 6\left(x-\frac{5}{12}+\frac{13}{12}\right)\left(x-\frac{5}{12}-\frac{13}{12}\right)$
$= 6\left(x+\frac{8}{12}\right)\left(x-\frac{18}{12}\right)$
$= 6\left(x+\frac{2}{3}\right)\left(x-\frac{3}{2}\right)$
$= (3x+2)(2x-3)$ //

5e $14+45x-14x^2$
$= -14\left(-1-\frac{45}{14}x+x^2\right)$
$= -14\left[\left(x-\frac{45}{28}\right)^2 - \frac{45^2}{28^2} - 1\right]$
$= -14\left[\left(x-\frac{45}{28}\right)^2 - \frac{2809}{784}\right]$
$= -14\left[\left(x-\frac{45}{28}+\frac{53}{28}\right)\left(x-\frac{45}{28}-\frac{53}{28}\right)\right]$
$= -14\left(x+\frac{2}{7}\right)\left(x-\frac{7}{2}\right)$
$= -(7x+2)(2x-7)$ //

f $12x^2+x-6$
$= 12\left[x^2+\frac{x}{12}-\frac{1}{2}\right]$
$= 12\left[\left(x+\frac{1}{24}\right)^2 - \frac{1}{24^2} - \frac{1}{2}\right]$
$= 12\left[\left(x+\frac{1}{24}+\frac{17}{24}\right)\left(x+\frac{1}{24}-\frac{17}{24}\right)\right]$
$= 12\left(x+\frac{3}{4}\right)\left(x-\frac{2}{3}\right)$
$= (4x+3)(3x-2)$ //

6b $x^2-3x+5=(x-\frac{3}{2})^2-\frac{9}{4}+5$

$(x+\frac{3}{2})^2+\frac{11}{4}$

min value $=\frac{11}{4}$ when $x=\frac{3}{2}$

d $2x^2-5x+2=2(x^2-\frac{5}{2}x)+2$

$=2[(x-\frac{5}{4})^2-\frac{25}{16}]+2$

$=2(x-\frac{5}{4})^2-\frac{25}{8}+\frac{16}{8}$

$=2(x-\frac{5}{4})^2-\frac{9}{8}$

Min value $=-\frac{9}{8}$ at $x=\frac{5}{4}$

e $3x^2+2x-4=3(x^2+\frac{2}{3}x)-4$

$=3[(x+\frac{1}{3})^2-\frac{1}{9}]-4$

$=3(x+\frac{1}{3})^2-\frac{1}{3}-4=3(x+\frac{1}{3})^2-\frac{13}{3}$

Least value $=-\frac{13}{3}$ at $x=-\frac{1}{3}$

f $3-7x-3x^2=3-3(x^2+\frac{7}{3}x)$

$=3-3[(x+\frac{7}{6})^2-\frac{49}{36}]$

$=3-3(x+\frac{7}{6})^2+\frac{49}{12}$

$=\frac{85}{12}-3(x+\frac{7}{6})^2$

Max value $=\frac{85}{12}$ at $x=-\frac{7}{6}$

7a. $f(x)=x^2-6x+10$

$=(x-3)^2-9+10$

$=(x-3)^2+1$

Range $f(x)\geq 1$

b $f(x)=x^2+7x+1$

$=(x+\frac{7}{2})^2-\frac{49}{4}+1$

$=(x+\frac{7}{2})^2-\frac{45}{4}$

$f(x)\geq -\frac{45}{4}$

c $f(x)=x^2-3x+4$

$=(x-\frac{3}{2})^2-\frac{9}{4}+4$

$=(x-\frac{3}{2})^2+1\frac{3}{4}$

Range. $f(x)\geq 1\frac{3}{4}$

8 b $y=x^2+6x-2$

$y=(x+3)^2-9-2$

$y=(x+3)^2-11$

vertex $(-3,-11)$

line of symmetry $x=-3$

c $y=7-10x-x^2$

$y=7-(x^2+10x)$

$y=7-[(x+5)^2-25]$

$y=32-(x+5)^2$

vertex $(-5,32)$

los eqn $x=-5$

d $y=x^2+3x+1$

$y=(x+\frac{3}{2})^2-\frac{9}{4}+1$

$y=(x+\frac{3}{2})^2-\frac{5}{4}$

vertex $(-\frac{3}{2},-\frac{5}{4})$

los eqn $x=-\frac{3}{2}$

e. $y=2x^2-7x+2$

$=2(x^2-\frac{7}{2}x+1)$

$=2[(x-\frac{7}{4})^2-\frac{49}{16}+1]$

$=2[(x-\frac{7}{4})^2-\frac{33}{16}]$

$=2(x-\frac{7}{4})^2-\frac{33}{8}$

i vertex $(\frac{7}{4},-\frac{33}{8})$

ii LOS : $x=\frac{7}{4}$

f. $y=3x^2-12x+5$

$=3(x^2-4x)+5$

$=3[(x-2)^2-4]+5$

$=3(x-2)^2-7$

i vertex $(2,-7)$

ii LOS : $x=2$

g $x > 0, x \in R$

! (a) $f(x) = (x+2)(x+1)$

line of symmetry :

$x_s = \frac{-2-1}{2} = -\frac{3}{2}$ → beyond domain

$f(0) = (0+2)(0+1) = 2$

Domain $x > 0$

Range : $f(x) > 2$

b $f(x) = (x-1)(x-2)$

line of symmetry :

$x_s = \frac{1+2}{2} = \frac{3}{2}$ → within domain

$= x^2 - 3x + 2$

$= (x-\frac{3}{2})^2 - \frac{9}{4} + 2$

$= (x-\frac{3}{2})^2 - \frac{1}{4}$

Range : $f(x) \geq -\frac{1}{4}$

c $f(x) = 2x^2 - 5x + 2 = 2(x^2 - \frac{5}{2}) + 2$

$= 2[(x-\frac{5}{4})^2 - \frac{25}{16}] + 2$

$= 2(x-\frac{5}{4})^2 - \frac{25}{8} + \frac{16}{8}$

$= 2(x-\frac{5}{4})^2 - \frac{9}{8}$

line of symmetry : $x_s = \frac{5}{4}$ → within domain

Range : $f(x) \geq -\frac{9}{8}$

10 a $y = x^2 - 4x - 5 = (x-2)^2 - 4 - 5$

$= (x-2)^2 - 9$

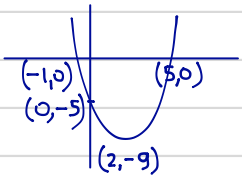
vertex $(2, -9)$

$y = (x-5)(x+1)$

x-axis intercept

$(5,0)$ & $(-1,0)$

y-axis intercept $(0,-5)$



e $y = 3x^2 - 12x + 9$

$y = 3(x^2 - 4x + 3)$

$= 3(x-1)(x-3)$

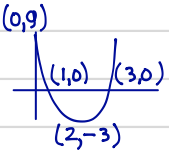
$y = 3[(x-2)^2 - 4] + 9$

$= 3(x-2)^2 - 3$

vertex : $(2, -3)$

x-axis intercept.

$(1,0)(3,0)$



11 a $y = (x-2)(x-4)$

$y = x^2 - 6x + 8$

b $y = (x-2)^2 - 6$

$y = x^2 - 4x + 4 - 6$

$y = x^2 - 4x - 2$

c $y = x^2 + bx + c$

$y = (x+1)^2 + p$

$(0,5)$ $5 = (0+1)^2 + p$

$5 = 1 + p$

$p = 4$

$y = x^2 + 2x + 1 + 4$

$y = x^2 + 2x + 5$

d $y = -20 - 8x - x^2$

$y = -(x^2 + 8x + 20)$

$y = -[(x+4)^2 - 16 + 20]$

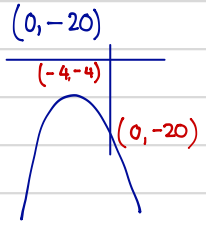
$y = -(x+4)^2 - 4$

Vertex $(-4, -4)$

x-axis intercept none

y-axis intercept

$(0, -20)$



f $y = 2x^2 - 5x + 1$

$y = 2(x^2 - \frac{5}{2}x) + 1$

$y = 2[(x-\frac{5}{4})^2 - \frac{25}{16}] + 1$

$y = 2(x-\frac{5}{4})^2 - \frac{25}{8} + \frac{8}{8}$

$y = 2(x-\frac{5}{4})^2 - \frac{17}{8}$

vertex $(\frac{5}{4}, -\frac{17}{8})$

$y = [(x-\frac{5}{4})\sqrt{2} - \sqrt{\frac{17}{8}}][(x-\frac{5}{4})\sqrt{2} + \sqrt{\frac{17}{8}}]$

$y = [\sqrt{2}x - \frac{5}{4}\sqrt{2} - \frac{1}{4}\sqrt{34}][\sqrt{2}x - \frac{5}{4}\sqrt{2} + \frac{1}{4}\sqrt{34}]$

x axis intercepts:

$\sqrt{2}x = \frac{5}{4}\sqrt{2} + \frac{1}{4}\sqrt{2}\sqrt{17}$

$x = \frac{5}{4} + \frac{1}{4}\sqrt{17}$

$\sqrt{2}x = \frac{5}{4}\sqrt{2} - \frac{1}{4}\sqrt{2\ 17}$

$x = \frac{5}{4} - \frac{1}{4}\sqrt{17}$