

## Exercise 4C

1 b  $x+y=7$       $x=7-y$

$$x^2 = y^2 + 25$$

$$(7-y)^2 = y^2 + 25$$

$$49 - 14y + y^2 = y^2 + 25$$

$$24 = 14y$$

$$y = \frac{24}{14} = \frac{12}{7}$$

$$x = 7 - \frac{12}{7} = \frac{37}{7}$$

2 b  $y = 3x+2$  }  
 $x^2 + y^2 = 26$  }

$$x^2 + (3x+2)^2 = 26$$

$$x^2 + 9x^2 + 12x + 4 = 26$$

$$10x^2 + 12x - 22 = 0$$

$$5x^2 + 6x - 11 = 0$$

$$(5x+11)(x-1) = 0$$

$$x = -\frac{11}{5}$$

$$y = 3\left(-\frac{11}{5}\right) + 2 = -\frac{33}{5} + 2$$

$$= -6\frac{3}{5} + 2 = -4\frac{3}{5}$$

$$\left(-\frac{11}{5}, -\frac{23}{5}\right)$$

$$x = 1$$

$$y = 3 + 2 = 5$$

$$(1, 5)$$

d  $x+2y=3 \Leftrightarrow x=3-2y$

$$x^2 + xy = 2$$

$$(3-2y)^2 + (3-2y)y - 2 = 0$$

$$9 - 12y + 4y^2 + 3y - 2y^2 - 2 = 0$$

$$2y^2 - 9y + 7 = 0$$

$$(2y-7)(y-1) = 0$$

$$y = \frac{7}{2}$$

$$x = 3 - 2\left(\frac{7}{2}\right) = -4 \quad \left. \right\} \left(-4, \frac{7}{2}\right)$$

$$y = 1$$

$$x = 3 - 2(1) = 1 \quad \left. \right\} (1, 1)$$

d  $y = 2-x$

$$x^2 - y^2 = 8$$

$$x^2 - (2-x)^2 = 8$$

$$x^2 - (4 - 4x + x^2) = 8$$

$$8x = 12$$

$$x = \frac{12}{8} = \frac{3}{2}$$

$$y = 2 - \left(\frac{3}{2}\right) = \frac{1}{2}$$

f  $y+2x=3 \Leftrightarrow y=3-2x$

$$2x^2 - 3xy = 14$$

$$2x^2 - 3x(3-2x) = 14$$

$$2x^2 - 9x + 6x^2 - 14 = 0$$

$$8x^2 - 9x - 14 = 0$$

$$(8x+7)(x-2) = 0$$

$$x = -\frac{7}{8}$$

$$y = 3 - 2\left(-\frac{7}{8}\right) = 3 + \frac{7}{4} = \frac{19}{4} \quad \left. \right\} \left(-\frac{7}{8}, \frac{19}{4}\right)$$

$$x = 2$$

$$y = 3 - 2(2) = -1 \quad \left. \right\} (2, -1)$$

f.  $y = 1-x$

$$y^2 - xy = 0$$

$$y(y-x) = 0$$

$$(1-x)(1-x-x) = 0$$

$$(1-x)(1-2x) = 0$$

$$x = 1, \quad x = \frac{1}{2}$$

$$y = 1-1 = 0 \quad y = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\left(-1, 0\right) \quad \left(\frac{1}{2}, \frac{1}{2}\right)$$

h  $y = 3x - 11$

$$x^2 + 2xy + 3 = 0$$

$$x^2 + 2x(3x-11) + 3 = 0$$

$$x^2 + 6x^2 - 22x + 3 = 0$$

$$7x^2 - 22x + 3 = 0$$

$$(7x-1)(x-3) = 0$$

$$x = \frac{1}{7}$$

$$y = 3x - 11 = \frac{3}{7} - 11 = -10\frac{4}{7}$$

$$x = 3$$

$$y = 9 - 11 = -2$$

h  $2x - 5y = 6 \Leftrightarrow 2x = 6 + 5y$

$$2xy - 4x^2 - 3y = 1$$

$$(2x)y - (2x)^2 - 3y - 1 = 0$$

$$(6+5y)y - (6+5y)^2 - 3y - 1 = 0$$

$$6y + 5y^2 - 36 - 60y - 25y^2 - 3y - 1 = 0$$

$$-20y^2 - 57y - 37 = 0$$

$$20y^2 + 57y + 37 = 0$$

$$(20y+37)(y+1) = 0$$

$$y = -\frac{37}{20}$$

$$2x = 6 + 5\left(-\frac{37}{20}\right) \quad \left(-\frac{13}{8}, -\frac{37}{20}\right)$$

$$x = 3 - \frac{37}{8} = -\frac{13}{8}$$

$$y = -1$$

$$2x = 6 + 5(-1) = 1$$

$$x = \frac{1}{2}$$

$$\left(\frac{1}{2}, -1\right)$$

$$\begin{aligned}
 3b \quad & \left. \begin{aligned} y &= \frac{1}{2}x - 1 \\ y &= 4x^2 \end{aligned} \right\} \\
 & \frac{1}{2}x - 1 = 4x^2 \\
 & 4x^2 - \frac{1}{2}x + 1 = 0 \\
 & D = \left(\frac{1}{2}\right)^2 - 4(4)(1) \\
 & D < 0 \Rightarrow \text{No intersection}
 \end{aligned}$$

$$\begin{aligned}
 4d \quad & x^4 - 5x^2 - 6 = 0 \\
 & \text{Let } x^2 = A \\
 & A^2 - 5A - 6 = 0 \\
 & (A-6)(A+1) = 0 \\
 & A = 6 \qquad A = -1 \\
 & x^2 = 6 \qquad [x^2 \text{ is never negative}] \\
 & x = \pm \sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 c. \quad & y = 3x - 1 \\
 & xy = 12 \\
 & x(3x - 1) = 12 \\
 & 3x^2 - x - 12 = 0 \\
 & D = (-1)^2 - 4(3)(-12) \\
 & D > 0 \Rightarrow 2 \text{ points of intersection}
 \end{aligned}$$

$$\begin{aligned}
 e. \quad & x^6 - 7x^3 - 8 = 0 \\
 & \text{Let } x^3 = A \\
 & A^2 - 7A - 8 = 0 \\
 & (A-8)(A+1) = 0 \\
 & A = 8 \qquad A = -1 \\
 & x^3 = 8 \qquad x^3 = -1 \\
 & x = \sqrt[3]{8} = 2 \qquad x = \sqrt[3]{-1} = -1
 \end{aligned}$$

$$\begin{aligned}
 f. \quad & 4y = 12 - x \Leftrightarrow x = 12 - 4y \\
 & xy = 9 \\
 & (12 - 4y)y = 9 \\
 & 12y - 4y^2 = 9 \\
 & 4y^2 - 12y + 9 = 0 \\
 & D = (-12)^2 - 4(4)(9) = 144 - 144 \\
 & D = 0 \Rightarrow 1 \text{ point of intersection}
 \end{aligned}$$

$$\begin{aligned}
 f \quad & x^6 + x^3 - 12 = 0 \\
 & A^2 + A - 12 = 0 \\
 & (A+4)(A-3) = 0 \\
 & A = -4 \qquad A = 3 \\
 & x^3 = -4 \qquad x^3 = 3 \\
 & x = \sqrt[3]{-4} \qquad x = \sqrt[3]{3}
 \end{aligned}$$

$$\begin{aligned}
 5c \quad & 2t + 5 = \frac{3}{t} \\
 & 2t^2 + 5t - 3 = 0 \\
 & (2t-1)(t+3) = 0 \\
 & t = \frac{1}{2}, -3
 \end{aligned}$$

$$\begin{aligned}
 l. \quad & 12x - 36 - 10x - 10 = -3(x+1)(x-3) \\
 & 2x - 46 = -3(x^2 - 2x - 3) \\
 & 3x^2 - 4x - 55 = 0 \\
 & (3x+11)(x-5) = 0 \\
 & x = -\frac{11}{3}, 5
 \end{aligned}$$

$$\begin{aligned}
 f \quad & \sqrt{t} = s \\
 & s^2 - 6s + 9 = 0 \\
 & (s-3)^2 = 0 \\
 & s = 3 \\
 & \sqrt{t} = 3 \\
 & t = 9
 \end{aligned}$$

$$\begin{aligned}
 l \quad & y^2 + 1 - y^2 = \frac{y^4 + y^2}{2} \\
 & 2 = y^4 + y^2 \\
 & A^2 + A - 2 = 0 \\
 & (A+2)(A-1) = 0 \\
 & A = -2 \qquad A = 1 \\
 & y^2 = -2 \qquad y^2 = 1 \\
 & \qquad \qquad \qquad y = \pm 1
 \end{aligned}$$

$$6c. t - 5\sqrt{t} - 14 = 0 \quad \sqrt{t} = x$$

$$x^2 - 5x - 14 = 0$$

$$(x-7)(x+2) = 0$$

$$x = 7 \quad x = -2$$

$$\sqrt{t} = 7 \quad \sqrt{t} = -2$$

$$t = \underline{49} \quad \text{square root never } \ominus$$

$$7 \quad y = mx - 1$$

$$y = x^2$$

$$mx - 1 = x^2$$

$$x^2 - mx + 1 = 0 \quad \begin{cases} a = 1 \\ b = -m \\ c = 1 \end{cases}$$

$$D = (-m)^2 - 4(1)(1) = 0$$

$$m^2 = 4$$

$$m = \underline{\pm 2}$$

$$6e. \sqrt[3]{x^2} - \sqrt[3]{x} - 6 = 0 \quad \sqrt[3]{x} = y$$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$$y = 3 \quad y = -2$$

$$\sqrt[3]{x} = 3 \quad \sqrt[3]{x} = -2$$

$$x = 27 \quad x = -8$$

$$8 \quad y = mx$$

$$x^2 + 2xy + 2x = 1$$

$$x^2 + 2x(mx) + 2x - 1 = 0$$

$$x^2 + 2mx^2 + 2x - 1 = 0$$

$$(1+2m)x^2 + 2x - 1 = 0 \quad \begin{cases} a = 1+2m \\ b = 2 \\ c = -1 \end{cases}$$

$$D = 4 - 4(1+2m)(-1) = 0$$

$$4 = -4(1+2m)$$

$$1+2m = -1$$

$$2m = -2$$

$$m = \underline{-1}$$

$$9 \quad y = 2x + k$$

$$3x^2 + y^2 = 12$$

$$3x^2 + (2x+k)^2 - 12 = 0$$

$$3x^2 + 4x^2 + 4kx + k^2 - 12 = 0$$

$$7x^2 + 4kx + k^2 - 12 = 0 \quad \begin{cases} a = 7 \\ b = 4k \\ c = k^2 - 12 \end{cases}$$

$$D = 16k^2 - 4(7)(k^2 - 12) = 0$$

$$= 16k^2 - 28k^2 + 336 = 0$$

$$12k^2 = 336$$

$$k^2 = \frac{336}{12} = 28$$

$$k = \pm \sqrt{28} = \pm \underline{2\sqrt{7}}$$