MISCELLANEOUS EX 4

3 i $2x^2-4x+1$

$= 2(x^2-2x)+1$

$= 2[(x-1)^2-1]+1$

$= 2(x-1)^2-1$

$y = 2(x-1)^2-1$

$A = (1,-1)$

ii $x-y+4=0$, $y=2x^2-4x+1$ } P & Q, $P(3,7)$

$y = x+4$

$2x^2-4x+1 = x+4$

$2x^2-5x-3=0$

$(2x+1)(x-3)=0$

$x=-\frac{1}{2}$ $\qquad$ $x=3$

$y=(-\frac{1}{2})+4$ $\qquad$ $y=(3)+4=7$

$Q(-\frac{1}{2},\frac{7}{2})$

iii $A(1,-1)$ $\quad$ $P(3,7)$

Midpoint of $AP \Rightarrow M(2,3)$

$Q(-\frac{1}{2},\frac{7}{2})$

$M(2,3)$

$m = \dfrac{\frac{1}{2}}{-\frac{5}{2}} = -\frac{1}{5}$, through $(2,3)$

$y-3=-\frac{1}{5}(x-2)$

$5y-15=-x+2$

$5y+x=17$

6 $A(-1,6)$ $\quad$ Mid-point of AB: $M(3,4)$

$B(7,2)$

$m_{AB} = \frac{4}{-8} = -\frac{1}{2}$

Perpendicular bisector of AB.

$m_{\perp}=2$, pass through $(3,4)$

$y-4=2(x-3)$

$\underline{y=2x-2}$

ii point $C(p,q)$ is on $y=2x-2$

$OC = 2$ units

$2^2 = p^2+q^2$ $\qquad$ $q=2p-2$ (2)

$p^2+q^2=4$ (1)

$p^2+(2p-2)^2=4$

$p^2+4p^2-8p+4=4$

$5p^2-8p=0$

$p(5p-8)=0$

$p=0$ $\qquad$ $p=\frac{8}{5}=1\frac{3}{5}$

$q=2p-2=-2$ $\qquad$ $q=2(\frac{8}{5})-2$

$(0,-2)$ $\qquad$ $=\frac{16}{5}-2=1\frac{1}{5}$

$(1\frac{3}{5}, 1\frac{1}{5})$

8 $A(a,2)$ $\quad$ $AB=\sqrt{125}$ units

$B(3,b)$ $\quad$ $m_{AB}=2$

$2=\dfrac{2-b}{a-3}$ $\qquad$ $125=(2-b)^2+(a-3)^2$

$2a-6=2-b$ $\qquad$ $125=4-4b+b^2+a^2-6a+9$

$b=8-2a$ (1) $\qquad$ $a^2+b^2-6a-4b-112=0$ (2)

(1) & (2) $\Rightarrow$ $a^2+(8-2a)^2-6a-4(8-2a)-112=0$

$a^2+64-32a+4a^2-6a-32+8a-112=0$

$5a^2-30a-80=0$

$a^2-6a-16=0$

$(a-8)(a+2)=0$

$a=8$ $\qquad$ $a=-2$

$b=8-2(8)=-8$ $\qquad$ $b=8-2(-2)=12$

10 a $6x^2+10x+5$

$=6\left(x^2+\frac{5}{3}x\right)+5$

$=6\left[\left(x+\frac{5}{6}\right)^2-\frac{25}{36}\right]+5$

$=6\left(x+\frac{5}{6}\right)^2-\frac{25}{6}+5$

$=\left(\sqrt{6}x+\frac{5}{\sqrt{6}}\right)^2-\frac{5}{6}$

$a=\sqrt{6},\ b=\frac{5}{\sqrt{6}},\ c=-\frac{5}{6}$

b $y\geqslant-\frac{5}{6}$

11 $y=x^2-4x+4$

$y=mx$

i $m=1 \Rightarrow y=x$

$\left.\begin{array}{l} y=x \\ y=x^2-4x+4 \end{array}\right\}$ A & B

$x=x^2-4x+4$

$x^2-5x+4=0$

$(x-4)(x-1)=0$

$x=4 \qquad x=1$

$y=4 \qquad y=1$

$A(4,4) \qquad B(1,1)$

Mid-point of AB :

$M=\left(\frac{5}{2},\frac{5}{2}\right)$

ii tangent to the curve $\Rightarrow D=0$

$mx=x^2-4x+4$

$x^2-4x-mx+4=0$

$x^2-x(4+m)+4=0$

$D=(4+m)^2-4(1)(4)=0$

$16+8m+m^2-16=0$

$m(m+8)=0$

$m=0,\ \underline{\underline{m=-8}}$

$-8x=x^2-4x+4$

$x^2+4x+4=0$

$(x+2)^2=0$

$x=-2$

$\left.\begin{array}{l} x=-2 \\ y=-8x=16 \end{array}\right\}(-2,16)$

12 $y=kx+6$

$y=x^2+3x+2k$

i $k=2$

$\left.\begin{array}{l} y=2x+6 \\ y=x^2+3x+4 \end{array}\right\}$ A & B

$2x+6=x^2+3x+4$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$\left.\begin{array}{l} x=-2 \\ y=2(-2)+6=2 \end{array}\right\}A(-2,2)$

$\left.\begin{array}{l} x=1 \\ y=2(1)+6=8 \end{array}\right\}B(1,8)$

Mid Point of AB

$M\left(-\frac{1}{2},5\right)$

ii $\left.\begin{array}{l} y=kx+6 \\ y=x^2+3x+2k \end{array}\right\}$ tangent $D=0$

$kx+6=x^2+3x+2k$

$x^2+3x-kx+2k-6=0$

$x^2+(3-k)x+2k-6=0$

$D=(3-k^2)-4(1)(2k-6)=0$

$9-6k+k^2-8k+24=0$

$k^2-14k+33=0$

$(k-11)(k-3)=0$

$k=11$ or $k=3$ //

13 $\left.\begin{array}{l} y=7\sqrt{x} \\ y=6x+k \end{array}\right\}$ A & B

i $k=2$:

$7\sqrt{x}=6x+2$

$6x-7\sqrt{x}+2=0$

$6A^2-7A+2=0$

$(2A-1)(3A-2)=0$

$A=\frac{1}{2} \qquad A=\frac{2}{3}$

$\sqrt{x}=\frac{2}{3}$

$x_A=\frac{1}{4} \qquad x_B=\frac{4}{9}$

ii $6x+k=7\sqrt{x}$ $\qquad D=0$

$6x-7\sqrt{x}+k=0$

$6A^2-7A+k=0$

$D=49-4(6)(k)=0$

$49=24k$

$\underline{\underline{k=\frac{49}{24}}}$

14 $y = ax^2 + 2bx + c$

$= a\left(x^2 + \frac{2b}{a}x\right) + c$

$= a\left[\left(x+\frac{b}{a}\right)^2 - \frac{b^2}{a^2}\right] + c$

$y = a\left(x+\frac{b}{a}\right)^2 - \frac{b^2}{a} + c$

i vertex : $\left(-\frac{b}{a}, -\frac{b^2}{a} + c\right) = \left(-\frac{b}{a}, c - \frac{b^2}{a}\right)$

ii ?

15.a $\left.\begin{array}{l} y = x-1 \\ y = kx^2 \end{array}\right\} \begin{array}{l} A \\ B \end{array}$

$x-1 = kx^2$

$kx^2 - x + 1 = 0$

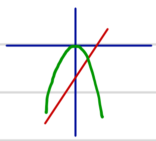
$D = 1-4k > 0$

$1 > 4k$

$k < \frac{1}{4}$

b if $k = \frac{1}{4} \rightarrow$ 1 point (tangent)

if $k > \frac{1}{4} \rightarrow$ No intersection



17a $(2,3)$ from $y = 2x+1$

$d^2 = (x-2)^2 + (y-3)^2$

$= (x-2)^2 + (2x+1-3)^2$

$= (x-2)^2 + (2x-2)^2$

$= x^2 - 4x + 4 + 4x^2 - 8x + 4$

$d^2 = 5x^2 - 12x + 8$

$= 5\left(x^2 - \frac{12}{5}x\right) + 8$

$= 5\left[\left(x-\frac{6}{5}\right)^2 - \frac{36}{25}\right] + 8$

$= 5\left(x-\frac{6}{5}\right)^2 - \frac{36}{5} + \frac{40}{5}$

$d^2_{min} = \frac{4}{5}$

$d_{min} = \sqrt{\frac{4}{5} \times \frac{5}{5}} = \frac{2}{5}\sqrt{5}$

b $(-1,3)$ from $y = -2x+5$

$d^2 = (x+1)^2 + (y-3)^2$

$d^2 = (x+1)^2 + (-2x+2)^2$

$= x^2 + 2x + 1 + 4x^2 - 8x + 4$

$= 5x^2 - 6x + 5$

$= 5\left(x^2 - \frac{6}{5}x + 1\right)$

$= 5\left[\left(x-\frac{3}{5}\right)^2 - \frac{9}{25} + 1\right]$

$= 5\left(x-\frac{3}{5}\right)^2 + \frac{16}{5}$

$d^2 min = \frac{16}{5}$

$d\, min = \frac{4}{5}\sqrt{5}$

16 $(1,2)$ to $y = 3x+5$

a d from $(1,2)$ to (x,y) :

$d = \sqrt{(x-1)^2 + (y-2)^2}$

$d^2 = (x-1)^2 + (y-2)^2$

$d^2 = (x-1)^2 + (3x+5-2)^2$

$= (x-1)^2 + (3x+3)^2$

$= x^2 - 2x + 1 + 9x^2 + 18x + 9$

$d^2 = 10x^2 + 16x + 10$

$= 10\left(x^2 + \frac{8}{5}x + 1\right)$

$= 10\left[\left(x+\frac{4}{5}\right)^2 - \frac{16}{25} + 1\right]$

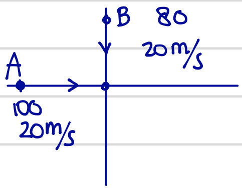
$d^2 = 10\left[\left(x+\frac{4}{5}\right)^2 + \frac{9}{25}\right]$

$d^2 = 10\left(x+\frac{4}{5}\right)^2 + \frac{18}{5}$

$d^2\ min = \frac{18}{5}$

$d_{min} = \sqrt{\frac{18}{5} \times \frac{5}{5}} = \frac{3}{5}\sqrt{10}$

18.



$$d^2 = (100-20t)^2 + (80-20t)^2$$
$$d^2 = [20(5-t)]^2 + [20(4-t)]^2$$
$$= 400[(5-t)^2 + (4-t)^2]$$
$$= 400[25 - 10t + t^2 + 16 - 8t + t^2]$$
$$= 400[2t^2 - 18t + 41]$$
$$= 400 \cdot 2\left[t^2 - 9t + \frac{41}{2}\right]$$
$$= 400 \cdot 2\left[\left(t-\frac{9}{2}\right)^2 - \frac{81}{4} + \frac{82}{4}\right]$$
$$= 800\left[\left(t-\frac{9}{2}\right)^2 + \frac{1}{4}\right]$$
$$d^2 \min = 200$$
$$d\min = \sqrt{200} = \underline{\underline{10\sqrt{2}}} \text{ m}$$

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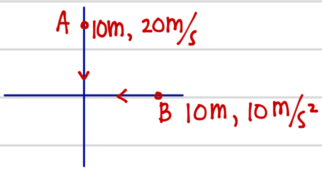

$$d^2 = (10-20t)^2 + (10-10t)^2$$
$$= [10(1-2t)]^2 + [10(1-t)]^2$$
$$= 100[(1-2t)^2 + (1-t)^2]$$
$$= 100(1 - 4t + 4t^2 + 1 - 2t + t^2)$$
$$= 100(5t^2 - 6t + 2)$$
$$= 100\ 5\left(t^2 - \frac{6}{5}t + \frac{2}{5}\right)$$
$$= 500\left[\left(t-\frac{3}{5}\right)^2 - \frac{9}{25} + \frac{10}{25}\right]$$
$$= 500\left(t-\frac{3}{5}\right)^2 + 500 \times \frac{1}{25}$$
$$d^2 \min = 20$$
$$d\min = \sqrt{20} = \underline{\underline{2\sqrt{5}}}$$

21

$$p = 100t - \frac{1}{2}t^2 - 200$$
$$= -\frac{1}{2}t^2 + 100t - 200$$
$$= -\frac{1}{2}(t^2 - 200t + 400)$$
$$= -\frac{1}{2}[(t-100)^2 - 10^4 + 400]$$
$$= -\frac{1}{2}[(t-100)^2 - 9\,600]$$
$$p_{max} = \frac{1}{2}(9\,600) = \$\,4800$$
$t = 100$ tonnes of cans