Score :

BUKIT SION HIGH SCHOOL

ACADEMIC YEAR 2019-2020 *Where the abundance in life flows……..*

Subject	**: Mathematics**	**Class**	**: 11.1 / 11.2 / 11.3 / 11.4**
Day / date	**:……………, ….. September 2019**	**Duration**	**: 90 minutes**
Topic	**: Functions, Quadratics, Inequalities**	**Name**	**: …………………………**

Instructions:

1. **Use black or dark blue ink. Don't use erasable pen.**
2. **Do not use correction tape/fluid.**
3. **Do not use calculator.**

1. Given $f(x) = 2x^2 + 3x - 1$ and $g(x) = 3x + 3$, find the values of
   a. $f(-2) + 2g(3)$ [3]
   b. a if $f(a) = -1$ [2]
   c. b if $f(b) = g(b)$ [3]

Answer:

1a. $f(-2) = 8 - 6 - 1 = 1$ [1]

$g(3) = 12$ [1]

$f(-2) + 2.g(3) = 1 + 2.12$ [1]

$= \underline{25}$

1b. $f(a) = -1$

$2a^2 + 3a - 1 = -1$ [1]

$2a^2 + 3a = 0$

$a(2a+3) = 0$

$a = 0, -\frac{3}{2}$ [1]

1c. $f(b) = g(b)$

$2b^2 + 3b - 1 = 3b + 3$ [1]

$2b^2 - 1 - 3 = 0$

$2b^2 = 4$

$b^2 = 2$ [1]

$b = \pm\sqrt{2}$ [1]

2. Find the range of each of the following functions for the given domain:
   a. $f(x) = 5x$ for $-1 \le x \le 3$ [2]
   b. $f(x) = x^2 - 5$ for $-1 < x < 3$ [2]

Answer:

2a. $f(-1) = -5$

$f(3) = 15$

$-5 \leqslant f(x) \leqslant 15$

2b. $f(-1) = -4$

$f(3) = 4$

$-5 \leqslant f(x) < 4$

3. Find the domain of the following functions:

   a. $f(x) = \sqrt{6 - 3x^2}$ [3]

   b. $f(x) = \frac{5}{x-8}$ [1]

Answer:

3a. $6 - 3x^2 \geqslant 0$ [1]

$3x^2 \leqslant 6$

$x^2 \leqslant 2$ [1]

$-\sqrt{2} \leqslant x \leqslant \sqrt{2}$ [1]

3b. $x - 8 \neq 0$

$x \neq 8,\ x \in R$ [1]

4. Sketch the following graphs. Your graph should show axes intersection.

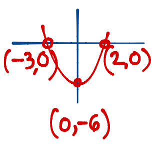
   a. $y = (x-2)(x+3)$ [3]

   b. $y = (2-x)(x+3)^2$ [3]

   c. $y = (x-1)(x+2)(x-3)$ [3]

Answer:

4a. ZV: $x = 2, -3$ [1]



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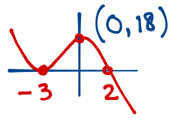
4b. ZV : $x = 2, -3$ [1]



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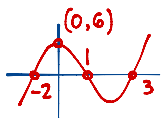
4c. ZV: $x = 1, -2, 3$ [1]



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[1]

5. Find the equation of the parabola (in the form $\boldsymbol{y = ax^2 + bx + c}$) which pass through points

   a. $(3, 0), (-2, 0)$ and $(0, 4)$. [4]

   b. $(3, 0)$ and has point $P(5, -2)$ as vertex. [4]

Answer:

5a. $y = a(x-3)(x+2)$

$(0,4)$: $4 = a(-3)(2)$ [1]

$4 = -6a$

$a = -\frac{4}{6} = -\frac{2}{3}$ [1]

$y = -\frac{2}{3}(x-3)(x+2)$ [1]

$y = -\frac{2}{3}(x^2 - x - 6)$

$y = -\frac{2}{3}x^2 + \frac{2}{3}x + 4$ [1]

5b. $y = a(x-5)^2 - 2$

$(3,0)$: $0 = a(-2)^2 - 2$ [1]

$0 = a \cdot 4 - 2$

$2 = 4a$

$a = \frac{1}{2}$ [1]

$y = \frac{1}{2}(x-5)^2 - 2$ [1]

$y = \frac{1}{2}(x^2 - 10x + 25) - 2$

$y = \frac{1}{2}x^2 - 5x + \frac{21}{2}$ [1]

6. Solve:

a. $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6 = 0$ [4]

b. $2x^2 - 5x \geq 3$ [4]

Answer:

6a. Let $x^{\frac{1}{3}} = y$

$y^2 + y - 6 = 0$ [1]

$(y+3)(y-2) = 0$ [1]

$y = -3$ or $y = 2$ [1]

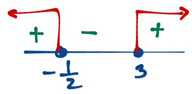
$x^{\frac{1}{3}} = -3$ $\quad$ $x^{\frac{1}{3}} = 2$

$x = (-3)^3 = -27$ $\quad$ $x = 2^3 = 8$ [1]

6b. $2x^2 - 5x - 3 \geq 0$

$(2x+1)(x-3) \geq 0$ [1]

ZV: $x = -\frac{1}{2}, 3$ [1]



$x \leq -\frac{1}{2}, \; x \geq 3$ [2]

7. A curve has equation $y = x^2 - 2x$ and a line has equation $y = kx - 4$, where k is a constant.

i. Show that the x-coordinates of points of intersection of the line and the curve are given by the equation $x^2 - (2+k)x + 4 = 0$ [1]

ii. For the case where the line intersects the curve at two distinct points, find the set value of k. [2]

iii. For the case where the line is a tangent to the curve at a point P, find the possible values of k and the related coordinates of P for each value of k. [4]

(i) $x^2 - 2x = kx - 4$

$x^2 - 2x - kx + 4 = 0$

$x^2 - (2+k)x + 4 = 0$ Shown!

(ii) Intersect at 2 distinct points:

$D > 0$

$(2+k)^2 - 4(1)(4) > 0$ [1]

$4 + 4k + k^2 - 16 > 0$

$k^2 + 4k - 12 > 0$

$(k-2)(k+6) > 0$

ZV: $k = -6, 2$

$k < -6$ or $k > 2$ [1]

iii $D = 0$

$k = 2$ or $k = -6$ [1]

a. If $k = -6$

$x^2 - (2-6)x + 4 = 0$

$x^2 + 4x + 4 = 0$

$(x+2)^2 = 0$

$x = -2$,

$y = -6(-2) - 4$

$= 8$

$(-2, 8)$

If $k = 2$

$x^2 - (2+2)x + 4 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$ [1]

$y = 2.2 - 4 = 0$ [1]

$(2, 0)$ [1]

8. Points A and B have coordinates $(-3, 2)$ and $(5, 6)$ respectively. Line l_1 is the perpendicular bisector of AB.

a. Show that the equation of line l_1 is $y = -2x + 6$ [2]

Line l_1 cuts the x-axis at C.

b. Find the coordinates of C. [2]

a. Mid Point of AB is $P(1,4)$ [1]

$m_{AB} = \frac{4}{8} = \frac{1}{2}$

$m_{\perp} = -2$ [1]

$l_1 : y - 4 = -2(x-1)$

$y = -2x + 2 + 4$

$y = -2x + 6$ (Shown!)

ii x-axis intercept : $y = 0$

$-2x + 6 = 0$ [1]

$6 = 2x$

$x = 3$

$\therefore C(3,0)$ [1]

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   a. $f(-3) + 3g(2)$ [3]
   b. a if $f(a) = -1$ [2]
   c. b if $f(b) = g(b)$ [3]

Answer:

1a. $f(-3) = 18 - 9 - 1 = 8$ [1]
$g(2) = 6 + 3 = 9$ [1]
$f(-3) + 3g(2)$
$= 8 + 3.9 = 35$ [1]

1b. $f(a) = 2a^2 + 3a - 1 = -1$ [1]
$2a^2 + 3a = 0$
$a(2a + 3) = 0$
$a = 0, -\frac{3}{2}$ [1]

1c. $f(b) = g(b)$
$2b^2 + 3b - 1 = 3b + 3$ [1]
$2b^2 - 1 - 3 = 0$
$2b^2 = 4$
$b^2 = 2$ [1]
$b = \pm\sqrt{2}$ [1]

2. Find the range of each of the following functions for the given domain:
   a. $f(x) = 6x$ for $-1 \le x \le 3$ [2]
   b. $f(x) = x^2 - 7$ for $-1 < x < 3$ [2]

Answer:

2a. $f(-1) = -6$
$f(3) = 18$
$-6 \le f(x) \le 18$
[1] [1]

2b. $f(-1) = -6$
$f(3) = 2$
$-7 \le f(x) < 2$
[1] [1]

3. Find the domain of the following functions:

a. $f(x) = \sqrt{4 - 2x^2}$ [3]

b. $f(x) = \frac{5}{x-7}$ [1]

Answer:

3a.

$4 - 2x^2 \geq 0$ [1]

$4 \geq 2x^2$

$x^2 \leq 2$ [1]

$-\sqrt{2} \leq x \leq \sqrt{2}$ [1]

3b. $x - 7 \neq 0$ [1]

$x \neq 7, x \in \mathbb{R}$

4. Sketch the following graphs. Your graph should show axes intersection.

a. $y = (x+2)(x-3)$ [3]

b. $y = (2+x)(x-3)^2$ [3]

c. $y = (x+1)(x-2)(x-3)$ [3]

Answer:

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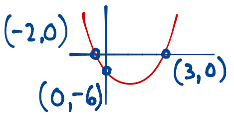

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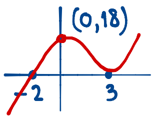

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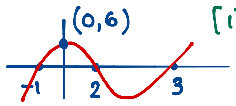
4c. ZV : $x = -1, 2, 3$ [1]



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b. $(3, 0)$ and has point $P(-5, -8)$ as vertex. [4]

Answer:

5a. $y = a(x+3)(x-2)$ [1]

$(0,4)$: $4 = a(3)(-2)$

$4 = -6a$

$a = -\frac{2}{3}$ [1]

$y = -\frac{2}{3}(x+3)(x-2)$ [1]

$y = -\frac{2}{3}(x^2 + x - 6)$

$y = -\frac{2}{3}x^2 - \frac{2}{3}x + 4$ [1]

5b. $y = a(x+5)^2 - 8$ [1]

$(3,0)$: $0 = a(8)^2 - 8$

$0 = 64a - 8$

$a = \frac{1}{8}$ [1]

$y = \frac{1}{8}(x+5)^2 - 8$ [1]

$y = \frac{1}{8}(x^2 + 10x + 25) - 8$

$y = \frac{1}{8}x^2 + \frac{5}{4}x + \frac{25}{8} - \frac{64}{8}$

$y = \frac{1}{8}x^2 + \frac{5}{4}x - \frac{39}{8}$ [1]

6. Solve:

a. $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$ [4]

b. $2x^2 - 5x \leq 3$ [4]

Answer:

6a.

Let $x^{\frac{1}{3}} = y$

$y^2 - y - 6 = 0$ [1]

$(y-3)(y+2) = 0$ [1]

$y = 3$ or $y = -2$ [1]

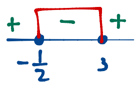
$x^{\frac{1}{3}} = 3$ $\quad$ $x^{\frac{1}{3}} = -2$

$x = 3^3 = 27$ $\quad$ $x = (-2)^3 = -8$ [1]

6b. $2x^2 - 5x - 3 \leq 0$

$(2x+1)(x-3) \leq 0$ [1]

zv: $x = -\frac{1}{2}$ or $x = 3$ [1]



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$k^2 + 4k - 12 > 0$

$(k-2)(k+6) > 0$

zv: $k = -6, 2$

$k < -6$ or $k > 2$ [1]

iii $D = 0$

$k = 2$ or $k = -6$ [1]

a. If $k = -6$

$x^2 - (2-6)x + 4 = 0$

$x^2 + 4x + 4 = 0$

$(x+2)^2 = 0$

$x = -2$,

$y = -6(-2) - 4$

$= 8$

$(-2, 8)$

If $k = 2$

$x^2 - (2+2)x + 4 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$ [1]

$y = 2.2 - 4 = 0$ [1]

$(2, 0)$ [1]

8. Points A and B have coordinates $(-3, 2)$ and $(5, 6)$ respectively. Line l_1 is the perpendicular bisector of AB.

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a. Mid Point of AB is $P(1,4)$ [1]

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$m_{\perp} = -2$ [1]

l_1 : $y-4=-2(x-1)$

$y=-2x+2+4$

$y=-2x+6$ (shown!)

ii x-axis intercept : $y=0$

$-2x+6=0$ [1]

$6=2x$

$x=3$

$\therefore C(3,0)$ [1]