

Miscellaneous Exercise 9

$$3a \quad (3a - 2b)^8$$

Coeff of $a^3 b^5$.

$$= \binom{8}{5} (3a)^3 (-2b)^5$$

$$= \frac{8!}{5!3!} (27)(-32) = \frac{8 \times 7 \times 6}{6} \times (-) =$$

$$4 \quad (3+5x)^7 = \binom{7}{0} 3^7 + \binom{7}{1} 3^6(5x) + \binom{7}{2} 3^5(5x)^2$$

$$= 3^7 + 7$$

$$5 \quad \left(2x - \frac{3}{x}\right)^5$$

$$i \quad = \binom{5}{0} (2x)^5 \left(-\frac{3}{x}\right)^0 + \binom{5}{1} (2x)^4 \left(-\frac{3}{x}\right)^1 + \binom{5}{2} (2x)^3 \left(-\frac{3}{x}\right)^2$$

$$= 32x^5 - 240x^3 + 720x$$

$$ii \quad \left(1 + \frac{2}{x^2}\right)(32x^5 - 240x^3 + 720x)$$

$$\text{Coeff of } x = 720x - 480x = 240x$$

$$\underline{240}$$

$$6i \quad (2-x)^6 = \binom{6}{0} (2)^6 (-x)^0 + \binom{6}{1} (2)^5 (-x)^1 + \binom{6}{2} (2)^4 (-x)^2$$

$$= 64 - 192x + 240x^2$$

$$ii \quad (1+2x+ax^2)(2-x)^6 = (1+2x+ax^2)(64-192x+240x^2)$$

$$\text{Coeff of } x^2 = 48$$

$$240 - 384 + 64a = 48$$

$$64a = 192$$

$$a = \underline{3}$$

$$7 \quad \left(x^2 + \frac{1}{x}\right)^3 = \binom{3}{0} (x^2)^3 \left(\frac{1}{x}\right)^0 + \binom{3}{1} (x^2)^2 \left(\frac{1}{x}\right)^1 + \binom{3}{2} (x^2) \left(\frac{1}{x}\right)^2 + \binom{3}{3} \left(\frac{1}{x}\right)^3$$

$$= x^6 + 3x^3 + 3 + \frac{1}{x^3}$$

$$9i \quad (2+3x)^5 = \binom{5}{0} 2^5 (3x)^0 + \binom{5}{1} 2^4 (3x) + \binom{5}{2} 2^3 (3x)^2$$

$$ii \quad (1+ax)(2+3x)^5 = (1+ax)(32+240x+720x^2+\dots)$$

$$\text{Coeff of } x^2 = 0$$

$$720 + 240a = 0$$

$$240a = -720$$

$$a = -\frac{720}{240} = \underline{-3}$$

$$13 \quad (p+q)^{10} = \binom{10}{0} p^{10} q^0 + \binom{10}{1} p^9 q^1 + \binom{10}{2} p^8 q^2 + \binom{10}{3} p^7 q^3$$

$$+ \binom{10}{4} p^6 q^4 + \binom{10}{5} p^5 q^5 + \dots$$

$$(2p-q)(p+q)^{10} = (\dots + \binom{10}{4} p^4 q^6 + \binom{10}{5} p^3 q^7 + \dots)$$

$$\text{Coeff of } p^4 q^7 = 2 \times \binom{10}{5} p^4 q^7 - \binom{10}{4} p^4 q^7$$

$$= 2 \times 252 p^4 q^7 - 210 p^4 q^7$$

$$= \underline{90} p^4 q^7$$

$$15. \quad \left(2x + \frac{1}{x^2}\right)^6$$

Term independent of x :

$$= \binom{6}{2} (2x)^4 \left(\frac{1}{x^2}\right)^2 = 15(16) = \underline{240}$$

$$17i \quad (2+u)^5 = 2^5 + \binom{5}{1} 2^4 u + \binom{5}{2} 2^3 u^2$$

$$= 32 + 80u + 80u^2$$

$$ii \quad (2+x+x^2)^5 = 32 + 80(x+x^2) + 80(x+x^2)^2$$

$$= 32 + 80x + 80x^2 + 80(x^2 + 2x^3 + x^4)$$

$$\text{Coeff of } x^2 = 80 + 80 = \underline{160}$$

$$19 \text{ i } (2+3x)^6 = 2^6 + \binom{6}{1}2^5(3x) + \binom{6}{2}2^4(3x)^2 \\ = 64 + 576x + 2160x^2$$

$$\text{ii } (1+ax)(2+3x)^6 = (1+ax)(64+576x+2160x^2)$$

$$\text{Coeff of } x^2 = 0$$

$$2160 + 576a = 0$$

$$576a = -2160$$

$$a = -\frac{15}{4}$$

$$20 \text{ i. } (2x-x^2)^6 = \binom{6}{0}(2x)^6(-x^2)^0 + \binom{6}{1}(2x)^5(-x^2)^1 + \binom{6}{2}(2x)^4(-x^2)^2 \\ = 64x^6 - 192x^7 + 240x^8$$

$$(2+x)(2x-x^2)^6 = (2+x)(64x^6 - 192x^7 + 240x^8)$$

$$\text{Coeff of } x^8: (2)(240)x^8 + (x)(-192x^7)$$

$$= (480 - 192)x^8 = \underline{288}x^8$$

$$23 \quad (7-6x)^3 + (7+6x)^3 = 1736$$

$$7^3 + 3(7)^2(-6x) + 3(7)(-6x)^2 + (-6x)^3 + 7^3 + 3(7)^2(6x) + 3(7)(6x)^2 + (6x)^3 = 1736$$

$$2 \times 7^3 + 2 \times 21 \times 36 x^2 = 1736$$

$$686 + 1512 x^2 = 1736$$

$$x^2 = \frac{25}{36}$$

$$x = \pm \frac{5}{6}$$

$$24. \left(2x^3 - \frac{1}{x^3}\right)^7$$

$$\text{Coeff of } x^6: \binom{7}{3}(2x^3)^4\left(-\frac{1}{x^3}\right)^3$$

$$= 35(16)x^{12}\left(-\frac{1}{x^9}\right)$$

$$= -\underline{560}x^6$$

$$21 \quad (1+3x+4x^2)^4 = (1+u)^4$$

$$= 1 + \binom{4}{1}u + \binom{4}{2}u^2 + \binom{4}{3}u^3 + \dots$$

$$= 1 + 4(3x+4x^2) + 6(3x+4x^2)^2 + \dots$$

$$= 1 + 12x + 16x^2 + 6(9x^2 + 24x^3 + 16x^4) + \dots$$

$$= 1 + 12x + 70x^2 + \dots$$

$$(1+3x+4x^2)^4 = 1.0304^4$$

$$3x+4x^2 = 0.0304$$

$$3x = 0.03 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} x = 0.01$$

$$4x^2 = 0.0004$$

$$1.0304^4 = 1 + 12 \times 0.01 + 0.007$$

$$= 1 + 0.12 + 0.007 = \underline{1.127}$$