

REV EX 1 p79

1. $A(4,8)$ or $B(10,26)$

$$m_{AB} = \frac{18}{6} = 3$$

$$y - 8 = 3(x - 4)$$

$$y = 3x - 12 + 8$$

$$y = 3x - 4 \text{ (shown!)}$$

$$\frac{y-8}{26-8} = \frac{x-4}{10-4}$$

$$\frac{y-8}{18} = \frac{x-4}{6}$$

$$6y - 48 = 18x - 72 \div 6$$

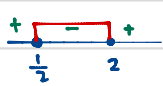
$$y - 8 = 3x - 12$$

$$y = 3x - 4 \text{ (shown!)}$$

5.a. $2x^2 - 5x + 2 \leq 0$

$$(2x-1)(x-2) \leq 0$$

2v: $x = \frac{1}{2}, 2$



$\therefore \frac{1}{2} \leq x \leq 2$

b. $(2x-3)^2 < 16$

$$-4 < 2x-3 < 4$$

$$-4+3 < 2x < 4+3$$

$$-\frac{1}{2} < x < \frac{7}{2}$$

c. $\frac{1}{3}x - \frac{1}{4}(2x-5) < \frac{1}{5}$

$$60 \left[\frac{1}{3}x - \frac{1}{4}(2x-5) \right] < 60 \times \frac{1}{5}$$

$$20x - 15(2x-5) < 12$$

$$20x - 30x + 75 < 12$$

$$63 < 10x$$

$$x > \frac{63}{10}$$

2. $5 + x - \sqrt{3+4x} = 0$

$$5+x = \sqrt{3+4x}$$

$$25 + 10x + x^2 = 3+4x$$

$$x^2 + 6x + 22 = 0$$

$$D = 36 - 4 \cdot 22 < 0 \rightarrow \text{No Solutions}$$

6. $2^{x+1} + 2^{x-1} = 160$

$$2 \cdot 2^x + \frac{1}{2} \cdot 2^x = 160$$

$$\left(2 + \frac{1}{2}\right) 2^x = 160$$

$$\frac{5}{2} \times 2^x = 160 \text{ (shown!)}$$

$$2^x = 160 \times \frac{2}{5} = 32 \times 2 = 2^6$$

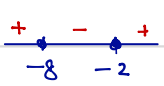
3. $x^2 + 10x + 38 = (x+5)^2 - 25 + 38$

$$= (x+5)^2 + 13 \quad b = 5, c = 13$$

a. Min value = 13 at $x = -5$

b. $x^2 + 10x + 38 \gg 22$

$$x^2 + 10x + 16 \gg 0$$

$$(x+2)(x+8) \gg 0$$


$$x \leq -8, x \geq -2$$

7. $\left. \begin{aligned} x^2 + 2xy + 2y^2 &= 5 \\ y &= 2x+k \end{aligned} \right\} D = 0$

$$x^2 + 2x(2x+k) + 2(2x+k)^2 = 5$$

$$x^2 + 4x^2 + 2kx + 2(4x^2 + 4kx + k^2) = 5$$

$$13x^2 + 10kx + 2k^2 - 5 = 0$$

$$D = 0$$

$$100k^2 - 4 \cdot 13(2k^2 - 5) = 0$$

$$100k^2 - 104k^2 + 260 = 0$$

$$-4k^2 + 260 = 0$$

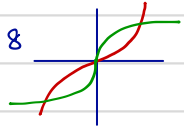
$$4k^2 = 260$$

$$k^2 = 65$$

$$k = \pm \sqrt{65}$$

4. $(4x^{\frac{1}{2}}y)^2 \div (2x^{-1}y^2)$

$$= \frac{16x^{\frac{1}{2}}y^2}{2x^{-1}y^2} = \underline{\underline{8x^{\frac{3}{2}}}}$$



$$x^3 - x^5$$

$$x^9 = x$$

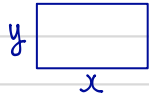
$$x(x^8 - 1) = 0$$

$$x(x^4 + 1)(x^4 - 1) = 0$$

$$x = 0, +1, -1$$

$$(0,0) (1,1) (-1,-1)$$

9 Frame - \$6/m perimeter
Glass \$15/m²



$$\text{Frame} = 2(x+y) \times 6$$

$$\text{Glass} = xy \times 15$$

$$\text{Total cost} = 12x + 12y + 15xy$$

$$12 = 12z + 12z + 15z^2$$

$$15z^2 + 24z - 12 = 0$$

$$5z^2 + 8z - 4 = 0$$

$$(5z-2)(z+2) = 0$$

$$z = \frac{2}{5} \text{ or } z = -2 \text{ (invalid)}$$

10. $B(2,-2) \rightarrow m_{BC} = \frac{2}{2} = 1$
 $C(4,0) \rightarrow BC = \sqrt{4+4} = 2\sqrt{2} \text{ unit}$
 $y - 0 = 1(x - 4)$
 $y = x - 4$

$A(1,4), m_{\perp} = -1$
 $y - 4 = -(x - 1)$
 $y = -x + 5$

Distance A to $y = x - 4$

$$d = \sqrt{(1-x)^2 + (4-y)^2}$$

$$d^2 = (1-x)^2 + (4-x+4)^2$$

$$d^2 = 1 - 2x + x^2 + 64 - 16x + x^2$$

$$= 2x^2 - 18x + 65 = 2(x^2 - 9x) + 65$$

$$= 2\left[\left(x - \frac{9}{2}\right)^2 - \frac{81}{4}\right] + 65 = 2\left(x - \frac{9}{2}\right)^2 - \frac{81}{2} + \frac{130}{2}$$

$$d^2_{\min} = \frac{49}{2}$$

$$d = \frac{7}{2}\sqrt{2}$$

$$\text{Area} = \frac{1}{2} \times \frac{7}{2}\sqrt{2} \times 2\sqrt{2} = 7 \text{ unit}^2$$

11. a. $2(3-x) < 4 - (2-x)$
 $6 - 2x < 4 - 2 + x$
 $4 < 3x$
 $x > \frac{4}{3}$

b. $(x-3)^2 < x^2$
 $x^2 - 6x + 9 < x^2$
 $9 < 6x$
 $x > \frac{9}{6}$
 $x > \frac{3}{2}$

c. $(x-2)(x-3) \geq 6$
 $x^2 - 5x + 6 \geq 6$
 $x^2 - 5x \geq 0$
 $x(x-5) \geq 0$
 zV: $x=0, x=5$

 $x \leq 0, x \geq 5$

12. $(p-1)x^2 + 4x + (p-4) = 0$
 Repeated root: $D = 0$
 $4^2 - 4(p-1)(p-4) = 0$
 $16 - 4(p^2 - 5p + 4) = 0$
 $16 - 4p^2 + 20p - 16 = 0$
 $-4p^2 + 20p = 0$
 $-4p(p-5) = 0$
 $p = 0 \text{ or } p = 5$

13. $2x + 3y = 5 \Leftrightarrow 3y = 5 - 2x$ ①

$x^2 + 3xy = 4 \Leftrightarrow x(x + 3y) = 4$ ②

① \rightarrow ② $x(x + 5 - 2x) = 4$

$x^2 + 5x - 2x^2 - 4 = 0$

$0 = x^2 - 5x + 4$

$(x-1)(x-4) = 0$

$x = 1$ or $x = 4$

$y = \frac{5-2}{3} = 1$ $y = \frac{5-8}{3} = -1$

14. $A(1,2)$ $m_{AB} = \frac{6}{8} = \frac{3}{4}$ } $AB \perp BC$
 $B(9,8)$ $m_{BC} = -\frac{4}{3}$
 $C(12,4)$

$AB = \sqrt{8^2 + 6^2} = 10$

$BC = \sqrt{3^2 + 4^2} = 5$

$Area = \frac{1}{2} \times 10 \times 5 = 25 \text{ unit}^2$

15. $y = 5 - 2x$ }
 $y = (3-x)^2$ }
 $5 - 2x = 9 - 6x + x^2$
 $x^2 - 4x + 4 = 0$
 $(x-2)^2 = 0$

$x = 2, y = 5 - 4 = 1$

$(2,1)$

\therefore line $y = 5 - 2x$ is a tangent to curve $y = (3-x)^2$

16. $A(-1,3)$ $C(5,-1)$ $O(2,1)$

$m_{AC} = \frac{4}{-6} = -\frac{2}{3}$

$m_{BD} = \frac{3}{2}$

Line AC:

$y - 3 = -\frac{2}{3}(x + 1)$

$y = -\frac{2}{3}x - \frac{2}{3} + 3$

$y = -\frac{2}{3}x + \frac{7}{3}$

Line BD:

$y - 1 = \frac{3}{2}(x - 2)$

$y = \frac{3}{2}x - 2$

$B(0,-2)$ $O(2,1)$

$D(4,4)$

17. $A(-1,2)$

$B(7,-4)$

a. $M(3,-1)$

b. $MB = \sqrt{16+9} = 5$

c. $P(2,y)$

$MB = PM$

$5^2 = 1^2 + (y+1)^2$

$25 = 1 + y^2 + 2y + 1$

$y^2 + 2y - 23 = 0$

$(y+1)^2 - 24 = 0$

$y+1 = \pm\sqrt{24}$

$y = -1 + \sqrt{24}$ or

$y = -1 - \sqrt{24}$

$\therefore y = -1 + \sqrt{24}$

$= -1 + 2\sqrt{6}$

18. a. $x^2 - x - 2 > 0$

$(x - \frac{1}{2})^2 - \frac{1}{4} - 2 > 0$

$(x - \frac{1}{2})^2 > \frac{9}{4}$

$x - \frac{1}{2} > \frac{3}{2}$ or $x - \frac{1}{2} < -\frac{3}{2}$

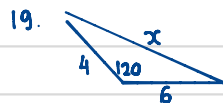
$x > 2$ or $x < -1$

b. $(x+1)(x-2)(x-3) > 0$

ZV: $x = -1, 2, 3$



$-1 < x < 2$ or $x > 3$



$x^2 = 4^2 + 6^2 + 2 \cdot 4 \cdot 6 \cos 120$

$= 16 + 36 + 48(-\frac{1}{2})$

$= 52 - 24 = 28$

$x = \sqrt{28} = 2\sqrt{7}$

20. $A(-3,2)$ $M(1,4)$

$C(5,6)$

$m_{AC} = \frac{4}{8} = \frac{1}{2}$

(i). $m_{\perp} = -2, M(1,4)$

$y - 4 = -2(x - 1)$

$y = -2x + 6$

$B(x,0)$

$0 = -2x + 6$

$x = 3$ $B(3,0)$

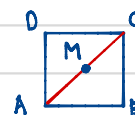
(ii). $A(-3,2)$ $B(3,0)$

$B(3,0)$ $C(5,6)$

$m_{AB} = \frac{2}{-6} = -\frac{1}{3}$ $m_{BC} = \frac{6}{2} = 3$

$-\frac{1}{3} \times 3 = -1$

$\therefore AB \perp BC$



(iii). $B(3,0)$ } $D(-1,8)$
 $M(1,4)$ }

$AD^2 = 2^2 + 6^2 = 4 + 36 = 40$

$AD = \sqrt{40} = 2\sqrt{10}$

$$21. AC = 2y + x = 16 \Rightarrow m = -\frac{1}{2}$$

$$B(2, 2) \quad m_{\perp} = 2$$

$$y - 2 = 2(x - 2)$$

$$(i). y = 2x - 2 \text{ \& } 2y + x = 16$$

$$(x = 16 - 2y)$$

$$y = 2(16 - 2y) - 2$$

$$y = 32 - 4y - 2$$

$$5y = 30$$

$$y = 6, x = 16 - 12 = 4$$

$$\therefore X(4, 6)$$

$$(ii). B(2, 2) \times (4, 6)$$

$$D(6, 10)$$

$$(iii). A(x, 0) \text{ is on } 2y + x = 16$$

$$0 + x = 16$$

$$x = 16 \quad A(16, 0)$$

$$C(0, y) \text{ is on } 2y + x = 16$$

$$2y = 16$$

$$y = 8 \quad C(0, 8)$$

$$AB = \sqrt{14^2 + 2^2} = \sqrt{200} = 10\sqrt{2}$$

$$CD = \sqrt{6^2 + 2^2} = \sqrt{40} = 2\sqrt{10}$$

$$\text{Perimeter} = 2(10\sqrt{2} + 2\sqrt{10})$$

$$= 40.9 \text{ unit}$$

$$22. \text{ vertex } (3, 4)$$

$$y = a(x - 3)^2 + 4$$

$$(-1, 0): 0 = a(-1 - 3)^2 + 4$$

$$0 = 16a + 4$$

$$16a = -4$$

$$a = -\frac{1}{4}$$

$$y = -\frac{1}{4}(x - 3)^2 + 4 //$$

$$(p, 0):$$

$$0 = -\frac{1}{4}(x - 3)^2 + 4$$

$$0 = -(x - 3)^2 + 16$$

$$(x - 3)^2 = 16$$

$$x - 3 = \pm 4$$

$$x_1 = 3 + 4 = 7 \quad (7, 0)$$

$$x_2 = 3 - 4 = -1 \quad (-1, 0)$$

$$(0, q):$$

$$q = -\frac{1}{4}(-3)^2 + 4$$

$$= -\frac{1}{4}(9) + 4$$

$$= \frac{7}{4} \quad (0, \frac{7}{4})$$

$$p = 7, q = \frac{7}{4}$$

$$23. y = 3^x$$

$$3^{2x+2} - 10 \times 3^x + 1 = 0$$

$$3^{2x} \times 3^2 - 10 \times 3^x + 1 = 0$$

$$9y^2 - 10y + 1 = 0$$

$$(9y - 1)(y - 1) = 0$$

$$y = \frac{1}{9} \quad \text{or} \quad y = 1$$

$$3^x = 3^{-2} \quad 3^x = 1$$

$$x = -2 \quad x = 0$$