Score :

BUKIT SION HIGH SCHOOL

ACADEMIC YEAR 2019-2020

Where the abundance in life flows……..

Subject	: **Mathematics**	**Class**	: **11.1 / 11.2 / 11.3 / 11.4**
Day / date	: **……………, ….. December 2019**	**Duration**	: **80 minutes**
Topic	: **1. Trigonometry**	**Name**	: **…………………………**
	2. Circular Measure		

Instructions:

1. **Use black or dark blue ink. Don't use erasable pen.**
2. **Do not use correction tape/fluid.**
3. **You may use calculator.**

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1. Fill in the blank.

   a. $\sin(400)° = \sin(\ 40\ )° = \sin(\ 140\ )°$, for $-180 \leq x \leq 180$ [2]

   b. $\cos(330)° = \cos(\ 30\ )° = \cos(-30)°$, for $-180 \leq x \leq 180$ [2]

   c. $\tan(-30)° = \tan(\ 330\ )° = \tan(150\ )°$, for $0 \leq x \leq 360$ [2]

   d. $\sin\left(\frac{\pi}{3}\right) = \cos(\ \frac{\pi}{6}\ ) = \cos(\ \frac{11\pi}{6}\ )$, for $0 \leq x \leq 2\pi$ [2]

   $= \sin\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$

2. Given: $y = 5 - 2\sin^2(2x + 30)°$

   Find the **maximum and the minimum value** of the function.

   In each case, give the **smallest positive values of *x*** (in degrees) at which they occur.

   $\sin^2(2x+30) \begin{array}{l} \nearrow 1 \\ \searrow 0 \end{array}$

   [6]

$\sin(2x+30) = 0$

① $2x+30 = 0 \pm k.360$

$2x = -30 \pm k.360$

$x = -15 \pm k.180$

② $2x+30 = 180 \pm k.360$

$2x = 150 \pm k.360$

$x = 75 \pm k.180$

$x = \underline{75}, 165$

$\sin(2x+30) = \pm 1$

$\sin(2x+30) = -1$

$2x+30 = -90 \pm k.360$

$2x = -120 \pm k.360$

$x = -60 \pm k.180$

$x = 120$

$\sin(2x+30) = 1$

$2x+30 = 90° \pm k.360$

$2x = 60 \pm k.360$

$x = \underline{30} \pm k.180$

If $\sin^2(2x+30) = 0$

$y = 5 - 2.(0) = 5$

If $\sin^2(2x+30) = 1$

$y = 5 - 2(1) = 3$

Answer:

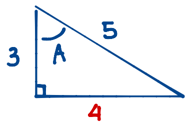
y_{max} = …5………..at x =…75°………

y_{min} = …3………..at x =…30°…………

3. Given $\cos A = \frac{3}{5}$ and $\sin A$ is negative. Find the value of:

   a. $\tan A$ (express your answer in exact value) [2]

   b. smallest positive value of A (in degrees, 1decimal place) [2]



$\cos A = \frac{3}{5}$, and $\sin A$ $\ominus$ $\Rightarrow$ A in Q4

a. $\tan A = -\frac{4}{3}$

b. $A = \tan^{-1}\left(-\frac{4}{3}\right) = -53.13 \pm k.180$

$A = 126.9, 306.9$

$\therefore A = 306.9^\circ$ (Q 4)

4. Find all the values of θ.

   a. $\sin(2\theta + 30)^\circ = -0.5$ for $0 \leq \theta \leq 180$ and give your answers correct to 1 decimal place. [4]

$2\theta + 30 = \sin^{-1}(-0.5)$

$2\theta + 30 = -30^\circ \pm k.360$ or $2\theta + 30 = 180 - (-30) \pm k.360$

$2\theta = -60 \pm k.360$

$\theta = -30 \pm k.180$

$= 150$

$2\theta = 180 \pm k.360$

$\theta = 90 \pm k.180$

$\therefore \theta = \underline{90^\circ, 150^\circ}$

   b. $\cos(\frac{3}{2}\theta - \overset{\frac{\pi}{9}}{\cancel{20}})^\circ = -0.5$ for $-\pi \leq \theta \leq \pi$ and give your answers correct to 2 decimal places. [6]

$\frac{3}{2}\theta - \frac{\pi}{9} = \frac{2\pi}{3} \pm k.2\pi$ or $\frac{3}{2}\theta - \frac{\pi}{9} = -\frac{2\pi}{3} \pm k.2\pi$

$\frac{3}{2}\theta = \frac{7\pi}{9} \pm k.2\pi$

$\theta = 1.63 \pm k.\frac{4\pi}{3}$

$= -2.56, 1.63$ rad

$\frac{3}{2}\theta = -\frac{5\pi}{9} \pm k.2\pi$

$\theta = -1.16 \pm k.\frac{4\pi}{3}$

$= -1.16, 3.03$ rad

$\therefore \theta = -2.56, -1.16, 1.63, 3.03$ // rad

$\frac{3}{2}\theta - 20 = 120 \pm k.360$ or $\frac{3}{2}\theta - 20 = -120 \pm k.360$

$\frac{3}{2}\theta = 140 \pm k.360$

$\theta = 93.3 \pm k.240$

$= -146.7^\circ, 93.3^\circ$

$\frac{3}{2}\theta = -100 \pm k.360$

$\theta = -66.7 \pm k.240$

$= -66.7^\circ, 173.3^\circ$

5. i. Show that equation $\frac{4\cos\theta}{\tan\theta} + 15 = 0$ can be written in the form

$$4sin^2\theta - 15\sin\theta - 4 = 0$$ [3]

ii. Hence solve the equation $\frac{4\cos\theta}{\tan\theta} + 15 = 0$ for $0° \leq \theta \leq 360°$ and give your answers correct to 1 decimal places. [3]

(i) $\frac{4\cos\theta}{\tan\theta} + 15 = 0$

$\frac{4\cos\theta}{\sin\theta/\cos\theta} = -15$ [1]

$4\cos^2\theta = -15\sin\theta$ [1]

$4(1-\sin^2\theta) = -15\sin\theta$ [1]

$0 = 4\sin^2\theta - 15\sin\theta - 4$

(shown!)

(ii) $4\sin^2\theta - 15\sin\theta - 4 = 0$

$(4\sin\theta + 1)(\sin\theta - 4) = 0$

$\sin\theta = -\frac{1}{4}$ or $\sin\theta = 4$ (Not possible)

$\theta = \sin^{-1}(-\frac{1}{4})$ [1]

$\theta = -14.5 \pm k.360$ or $\theta = 180-(-14.5) \pm k.360$ [1]

$\therefore \theta = \underline{\underline{194.5}}, \underline{\underline{345.5}}$ [1]

6. Prove the following identities:

$$\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta} \equiv \frac{4}{\sin\theta\tan\theta}$$ [4]

$\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta} \equiv \frac{4}{\sin\theta\tan\theta}$ [1]

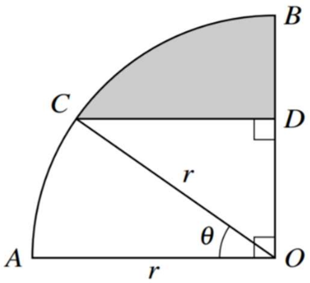
$\frac{(1+\cos\theta)^2 - (1-\cos\theta)^2}{1-\cos^2\theta} \equiv \frac{4}{\sin\theta\tan\theta}$ [1]

$\frac{1+2\cos\theta+\cos^2\theta - (1-2\cos\theta+\cos^2\theta)}{\sin^2\theta} \equiv \frac{4}{\sin\theta\tan\theta}$ [1]

$\frac{4\cos\theta}{\sin^2\theta} \equiv \frac{4}{\sin\theta\tan\theta}$ [1]

$\frac{4}{\sin\theta . \frac{\sin\theta}{\cos\theta}} \equiv \frac{4}{\sin\theta\tan\theta}$ (shown!)

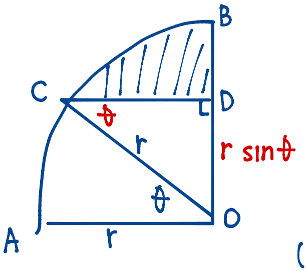
7. In the diagram, AOB is a quarter circle with centre O and radius r. The point C lies on the arc AB and the point D lies on OB. The line CD is parallel to O and angle $AOC = \theta$ radians.



(i) Express the perimeter of the shaded region in terms of r, θ and π. [4]

(ii) For the case where $r = 5$ cm and $\theta = 0.6$ rad, find the area of the shaded region. [3]

7.



$BD = r - r\sin\theta$ [1]

$CD = r\cos\theta$ [1]

$BC = \left(\frac{\pi}{2} - \theta\right) r$ [1]

(i) Perimeter $BCD = r - r\sin\theta + r\cos\theta + \left(\frac{\pi}{2} - \theta\right)r$ [1]

(ii). $r = 5, \theta = 0.6$ rad

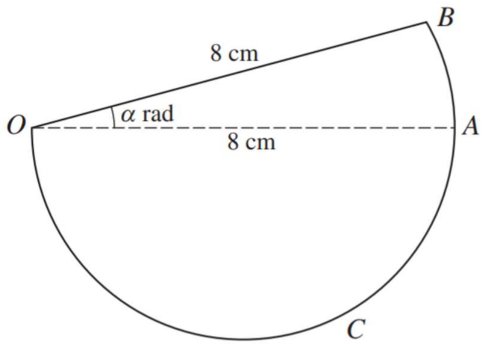
$OCB = \frac{\left(\frac{\pi}{2} - \theta\right)}{2} \times r^2 = \frac{\frac{\pi}{2} - 0.6}{2} \times 25 = 12.13$ [1]

$OCD = \frac{1}{2} OD \times CD = \frac{1}{2} r\sin\theta . r\cos\theta$

$= \frac{1}{2} . 5^2 \sin 0.6 \times \cos 0.6 = 5.83$ [1]

$BCD = 12.1 - 5.8 = \underline{\underline{6.3}}$ cm^2 [1]

8. In the diagram, OAB is a sector of a circle with centre O and radius 8 cm. Angle BOA is α radians. OAC is a semicircle with diameter OA. The area of the semicircle OAC is twice the area of the sector OAB.



Find:

(i) α in terms of π. [3]

(ii) the perimeter of the complete figure in terms of π. [2]

$OAC = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi 16 = 8\pi$ [1]

$OAB = \frac{\alpha}{2} 64 = 32\alpha$ [1]

$8\pi = 2 \times 32\alpha$

(i) $\alpha = \frac{8\pi}{64} = \frac{\pi}{8}$ rad [1]

(ii) $AB = \alpha r = \frac{\pi}{8} \times 8 = \pi$ [1]

$OCA = 4\pi$

Perimeter $= 8 + \pi + 4\pi = 5\pi + 8$ cm [1]

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1. Fill in the blank.

a. $\sin(420)^\circ = \sin(\ 60\ )^\circ = \sin(\ 120\ )^\circ$, for $-180 \le x \le 180$ [2]

b. $\cos(300)^\circ = \cos(\ 60\ )^\circ = \cos(-60)^\circ$, for $-180 \le x \le 180$ [2]

c. $\tan(-60)^\circ = \tan(\ 120\ )^\circ = \tan(\ 300\ )^\circ$, for $0 \le x \le 360$ [2]

d. $\cos\left(\frac{\pi}{3}\right) = \sin(\ \frac{\pi}{6}\ ) = \sin(\ \frac{5\pi}{6}\ )$, for $0 \le x \le 2\pi$ [2]

$\cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$

2. Given: $y = 5 - 2\cos^2(2x + 30)^\circ$

Find the **maximum and the minimum value** of the function.

In each case, give the **smallest positive values of x** (in degrees) at which they occur. [6]

$\cos^2(2x+30) < \begin{matrix} 1 \\ 0 \end{matrix}$

If $\cos^2(2x+30) = 0$, $y = 5-2(0) = 5$

$\cos(2x+30) = 0$

① $2x+30 = 90 \pm k.360$

$2x = 60 \pm k.360$

[1] $x = 30 \pm k.180$

② $2x+30 = -90 \pm k.360$

$2x = -120 \pm k.360$

$x = -60 \pm k.180$

If $\cos^2(2x+30) = 1$, $y = 5-2(1) = 3$

$\cos(2x+30) = \pm 1$

$\cos(2x+30) = 1$

$2x+30 = 0 \pm k.360$

[1] $2x = -30 \pm k.360$

$x = -15 \pm k.180$

$\cos(2x+30) = -1$

① $2x+30 = 180 \pm k.360$

$2x = 150 \pm k.360$

$x = 75 \pm k.180$

Answer:

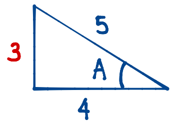
$y_{max} =$ ……5……at $x =$……30…… [2]

$y_{min} =$ ……3……at $x =$……75…… [2]

3. Given $\cos A = \frac{4}{5}$ and $\sin A$ is negative. Find the value of:

   a. $\tan A$ (express your answer in exact value) [2]

   b. smallest positive value of A (in degrees, 1decimal place) [2]



$\cos A = \frac{4}{5}$, $\sin A \ominus \Rightarrow A$ is in Q4

a. $\tan A = -\frac{3}{4}$

b. $A = \tan^{-1}\left(-\frac{3}{4}\right) = -36.9 \pm k.180$

$A = 143.13, 323.13$

$\therefore A = 323.13$ // (Q4)

4. Find all the values of θ.

   a. $\sin(2\theta - 30)° = -0.5$ for $0 \le \theta \le 180$ and give your answers correct to 1 decimal place. [4]

① $2\theta - 30 = -30 \pm k.360$ [1]

$2\theta = 0 \pm k.360$

$\theta = 0, 180$

② $2\theta - 30 = 180 - (-30) \pm k.360$ [1]

$2\theta = 240 \pm k.360$

$\theta = 120 \pm k.180$

$\therefore \theta = 0, 120, 180$ [2]

   b. $\cos(\frac{3}{2}\theta + \overset{\frac{\pi}{9}}{\cancel{20}})° = -0.5$ for $-\pi \le \theta \le \pi$ and give your answers correct to 2 decimal places. [6]

$\cos\left(\frac{3}{2}\theta + \frac{\pi}{9}\right) = -0.5$

① $\frac{3}{2}\theta + \frac{\pi}{9} = \frac{2\pi}{3} \pm k.2\pi$

$\frac{3}{2}\theta = \frac{2\pi}{3} - \frac{\pi}{9} \pm k.2\pi$

$\frac{3}{2}\theta = \frac{5\pi}{9} \pm k.2\pi$

$\theta = \frac{10}{27}\pi \pm k.\frac{4}{3}\pi$

$\theta = \frac{10}{27}\pi, -\frac{26}{27}\pi$

② $\frac{3}{2}\theta + \frac{\pi}{9} = -\frac{2\pi}{3} \pm k.2\pi$

$\frac{3}{2}\theta = -\frac{2\pi}{3} - \frac{\pi}{9} \pm k.2\pi$

$\frac{3}{2}\theta = -\frac{7\pi}{9} \pm k.2\pi$

$\theta = -\frac{14}{27}\pi \pm k.\frac{4\pi}{3}$

$\theta = -\frac{14}{27}\pi, \frac{22}{27}\pi$

$\therefore \theta = -\frac{26}{27}\pi, -\frac{14}{27}\pi, \frac{10}{27}\pi, \frac{22}{27}\pi$ or $\theta = -3.03, -1.63, 1.16, 2.56$ rad //

5. i. Show that equation $\frac{4\cos\theta}{\tan\theta}+15=0$ can be written in the form

$$4sin^2\theta - 15\sin\theta - 4 = 0$$ [3]

ii. Hence solve the equation $\frac{4\,co}{\tan\theta}+15=0$ for $0° \leq \theta \leq 360°$ and give your answers correct to 1 decimal places. [3]

6. Prove the following identities:

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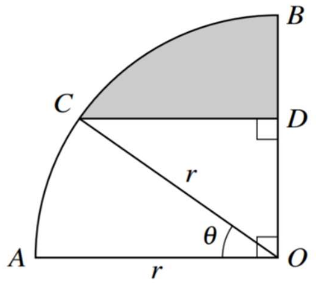

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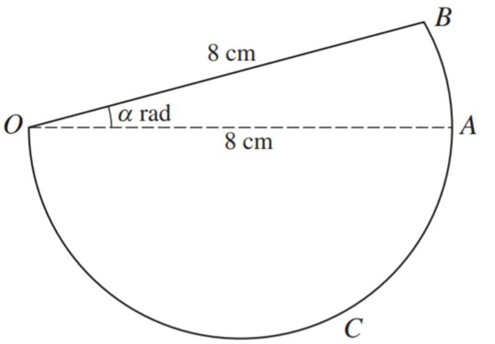

Find:

(i) α in terms of π. [3]

(ii) the perimeter of the complete figure in terms of π. [2]