

Exercice 14A

1d. $r = -3; 162, 486$

e. $r = 1.1; 1.4641, 1.61051$

f. $r = \frac{1}{x}; \frac{1}{x}, \frac{1}{x^2}$

2d. $U_i = 81 \cdot \left(\frac{1}{3}\right)^{i-1}$

e. $U_i = x \cdot x^{i-1} = x^i$

f. $U_i = p q_h^2 \cdot \left(\frac{q_h}{p}\right)^{i-1} = p \cdot q_h^{2+i-1} \cdot p^{1-i}$
 $= p^{2-i} \cdot q_h^{1+i}$

3a. $a = 2, r = 2, U_n = 2048$

$$2048 = 2 \cdot 2^{n-1} = 2^{1+n-1} = 2^n$$

$$n = \frac{\log 2048}{\log 2} = 11$$

3c. $r = 3, a = 2$

$$U_n = 1458 = 2 \times 3^{n-1}$$

$$3^{n-1} = 729 = 3^6$$

$$n-1 = 6$$

$$n = 7$$

d. $a = 5, r = -2$

$$U_n = -40960 = 5 \times (-2)^{n-1}$$

$$(-2)^{n-1} = -8192 = (-2)^{13}$$

$$n-1 = 13$$

$$n = 14$$

4a. $U_2 = 4 = a \cdot r$

$$U_5 = 108 = a \cdot r^4$$

$$108 = a \cdot r \times r^3$$

$$108 = 4 \times r^3$$

$$r^3 = \frac{108}{4} = 27$$

$$r = \sqrt[3]{27} = 3$$

4d. $U_3 = 8, U_9 = 64$

$$\frac{U_9}{U_3} = \frac{a r^8}{a r^2} = \frac{64}{8}$$

$$r^6 = 8$$

$$r^6 = 2^3$$

$$(r^6)^{\frac{1}{6}} = (2^3)^{\frac{1}{6}}$$

$$r = 2^{\frac{1}{2}} = \sqrt{2}$$

$$8 = a(\sqrt{2})^2$$

$$8 = a \times 2$$

$$a = 4$$

e. $\frac{U_{n+4}}{U_n} = \frac{a r^{n+3}}{a r^{n-1}} = \frac{40353607}{16807}$

$$r^4 = 2401 = (\pm 7)^4$$

$$r = \pm 7$$

$$a(\pm 7)^{n-1} = 16807$$

$$a = \frac{16807}{(\pm 7)^{n-1}}$$

5a. $a = 2, r = 3, n = 10$

$$S_n = \frac{2(3^{10} - 1)}{3 - 1} = 59048$$

5e. $3 + 6 + 12, n = 12$

$$r = 2, a = 3$$

$$S_{12} = \frac{a(r^n - 1)}{r - 1} = \frac{3(2^{12} - 1)}{1} = 12285$$

f. $a = 12, r = -\frac{1}{3}, n = 10$

$$S_{10} = \frac{12[1 - (-\frac{1}{3})^{10}]}{1 + \frac{1}{3}} = 0.75$$

6a. $1 + 2 + 4 + \dots + 1024$

$$a = 1, r = 2, U_n = 1024$$

$$1024 = 2^{n-1}$$

$$n - 1 = 10$$

$$n = 11$$

$$S_n = \frac{2^{11} - 1}{1} = 2047$$

Another Method (if U_n given)

$$S_n = \frac{n \cdot U_n - U_1}{r - 1} = \frac{2 \times 1024 - 1}{2 - 1} = \underline{2047}$$

6b. $1 - 2 + 4 - \dots + 1024$

$$a = 1, r = -2, U_n = 1024$$

$$S_n = \frac{(-2)(1024) - 1}{-2 - 1} = \frac{2049}{3} = 683$$

6j. $81 - 27 + 9 - \dots + \frac{1}{(-3)^n}$

$$a = 81, r = -\frac{1}{3}, U_n = \frac{1}{(-3)^n}$$

$$S_n = \frac{(-\frac{1}{3})(-\frac{1}{3})^n - 81}{-\frac{1}{3} - 1}$$

$$= -\frac{3}{4} \left[\left(-\frac{1}{3}\right) \left(-\frac{1}{3}\right)^n - 81 \right]$$

$$= \frac{1}{4} \left(-\frac{1}{3}\right)^n + \frac{243}{4}$$

10. $x, y, z = G_1, G_2, G_3$ of geo seq
 $= A_1, A_2, A_4$ of arith seq

$$y = rx \quad y = d + x$$

$$z = r^2x \quad z = x + 3d$$

$$rx = d + x \quad z = r^2x = x + 3d$$

$$rx - x = d$$

$$x(r-1) = d$$

$$x = \frac{d}{r-1}$$

$$\frac{r^2 d}{r-1} = \frac{d}{r-1} + 3d$$

$$\frac{r^2}{r-1} = \frac{1}{r-1} + 3$$

$$\frac{r^2}{r-1} = \frac{1+3r-3}{r-1}$$

$$r^2 = 3r - 2$$

$$r^2 - 3r + 2 = 0$$

$$(r-1)(r-2) = 0$$

$$r = 1, r = \underline{2}$$

$$y = rx = 2x = d + x$$

$$x = d$$

$$G_4 = x r^3 = 8x = x + (n-1)d$$

$$7x = (n-1) \cdot x$$

$$7x = nx - x$$

$$8x = nx$$

$$n = \underline{8}$$

11 $x, y, z \Rightarrow G_1, G_2, G_3$
 A_1, A_2, A_5

a $y = xr = x + d$

$$xr - x = d$$

$$x(r-1) = d$$

$$x = \frac{d}{r-1}$$

$$z = xr^2 = x + 4d$$

$$xr^2 - x = 4d$$

$$x(r^2-1) = 4d$$

$$\frac{d}{r-1} (r+1)(r-1) = 4d$$

$$d(r+1) = 4d$$

$$r+1 = 4$$

$$\underline{r = 3}$$

b $y = xr = x + d$

$$3x = x + d$$

$$2x = d$$

$$x = \frac{d}{2} //$$

$$G_4 = r^3x = 27x = x + (n-1)d$$

$$26x = (n-1)(2x)$$

$$26x = 2nx - 2x$$

$$28x = 2nx$$

$$n = \underline{14}$$