

Exercise 14 B

CONVERGENT SEQUENCES

$$S_n = \frac{a(1-r^n)}{1-r}$$

If $|r| < 1$, then when $n \rightarrow$ large
 $r^n \rightarrow$ small

$$S_\infty = \frac{a}{1-r}$$

1a. $S_\infty = \frac{1}{1-\frac{1}{2}} = 2$

h. $a = \frac{1}{2}, r = -\frac{1}{2}$

$$S_\infty = \frac{\frac{1}{2}}{1+\frac{1}{2}} = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$$

j. $a = 50, r = \frac{1}{5}$

$$S_\infty = \frac{50}{1-\frac{1}{5}} = 50 \times \frac{5}{4} = 625$$

4. $a = 11, S_\infty = 6$

$$S_\infty = \frac{a}{1-r}$$

$$6 = \frac{11}{1-r}$$

$$1-r = \frac{11}{6}$$

$$r = 1 - \frac{11}{6} = \frac{6}{6} - \frac{11}{6} = -\frac{5}{6}$$

5. $r = \frac{3}{4}, S_\infty = 12$

$$12 = \frac{a}{1-\frac{3}{4}}$$

$$12 \times \frac{1}{4} = a = \underline{3}$$

6. $r = -\frac{3}{5}, S_\infty = 12$

$$12 = \frac{a}{1+\frac{3}{5}}$$

$$12 = \frac{a}{8/5}$$

h. $a = 0.857142, r = 0.000001$

$$S_\infty = \frac{0.857142}{0.999999} = \frac{6}{7}$$

$$a = 12 \times \frac{8}{5} = \frac{96}{5} = \underline{19.2}$$

3. $a = 5, S_\infty = 6$

$$6 = \frac{5}{1-r}$$

$$6 - 6r = 5$$

$$1 = 6r$$

$$r = \frac{1}{6}$$

$$7 \quad 1 - \frac{1}{2} + \frac{1}{4} - \dots = \frac{2}{3} \text{ m}$$

$$1 + \left| -\frac{1}{2} \right| + \left| \frac{1}{4} \right| + \left| -\frac{1}{8} \right| + \dots$$

$$a = 1, r = \frac{1}{2}$$

$$S_{\infty} = \frac{1}{1 - \frac{1}{2}} = \underline{2 \text{ m}}$$

$$9. \quad S_{\infty} = 2 + 0.8 \times 2 + \dots$$

$$= \frac{2}{1 - 0.8} = \frac{2}{0.2} = 10 \text{ s}$$

$$10. \quad 1 + 0.9 \times 2 + 0.9^2 \times 2 + \dots$$

$$1 + 2 \times \frac{0.9}{1 - 0.9} = 1 + 2 \times 9 = \underline{19 \text{ m}}$$

$$11 \quad a = 1, r = \frac{1}{2}$$

$$S_{\infty} = \frac{1}{1 - \frac{1}{2}} = 2 \text{ m}$$

$$200 - 1 = 199 \text{ cm}$$

$$S_n = 1 + \frac{1}{2} + \frac{1}{4} + \dots = 1.99 \text{ m}$$

$$199 = \frac{1 \left(1 - \left(\frac{1}{2} \right)^n \right)}{\frac{1}{2}} = 2 \left(1 - \left(\frac{1}{2} \right)^n \right)$$

$$\frac{1.99}{2} = 1 - \left(\frac{1}{2} \right)^n$$

$$\left(\frac{1}{2} \right)^n = \frac{1}{200}$$

$$\frac{1}{2} \log_{\frac{1}{200}} = n = 7.64 \approx 8$$