Miscellaneous Exercise 14

1 $G_5 = 100 = ar^4$

$G_7 = 400 = ar^6$

$\frac{G_7}{G_5} = \frac{400}{100} = \frac{ar^6}{ar^4}$

$4 = r^2$

$r = \pm 2$

$G_5 = 100 = a(2)^4$

$100 = 16a$

$a = \frac{100}{16} = \frac{25}{4}$ //

2 $G_1 = a$, $r = \frac{1}{\sqrt{2}}$

$S_\infty = \frac{a}{1-\frac{1}{\sqrt{2}}} = \frac{a\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$= \frac{2a + a\sqrt{2}}{1} = a(2+\sqrt{2})$

3 $U_n = ar^{n-1}$

$a = 3$, $U_2 = -\frac{3}{4}$

$r = \frac{U_2}{U_1} = \frac{-\frac{3}{4}}{3} = -\frac{1}{4}$ //

$S_n = \frac{a(1-r^n)}{1-r} = \frac{3\left(1-\left(-\frac{1}{4}\right)^n\right)}{1+\frac{1}{4}}$

$= 3 \times \frac{4}{5}\left(1-\left(-\frac{1}{4}\right)^n\right)$

$= \frac{12}{5} - \frac{12}{5}\left(-\frac{1}{4}\right)^n$

4. $0.99 + 0.99^2 + 0.99^3 + \dots + 0.99^{99}$

$a = 0.99$, $r = 0.99$, $n = 99$

$S_{99} = \frac{0.99(1-0.99^{99})}{1-0.99}$

$\approx \frac{0.99}{0.01} \times 0.63 \approx 62$

5. $a = \frac{1}{10^3}$, $r = \frac{1}{10^3}$

$S_\infty = \frac{\frac{1}{10^3}}{1-\frac{1}{10^3}} = \frac{0.001}{0.999} = \frac{1}{999}$

$0.108108\dots = \frac{108}{999} = \frac{4}{37}$

6 $a = 1$, r

$S_\infty = 5 = \frac{1}{1-r}$

$5 - 5r = 1$

$4 = 5r$

$r = \frac{4}{5}$

$S_n > 4.9$

$\frac{a(1-r^n)}{1-r} > 4.9$

$\frac{1-\left(\frac{4}{5}\right)^n}{1-\frac{4}{5}} > 4.9$

$1-\left(\frac{4}{5}\right)^n > 4.9 \times 0.2$

$1 - 0.98 > \left(\frac{4}{5}\right)^n$

$0.02 > 0.8^n$

$\log 0.02 > n \log 0.8$

$\log(2\times10^{-2}) > n(\log 8 \times 0.1)$

$-2 + \log 2 > n(-1 + \log 8)$

$n > 17.53$

$n = 18$

7 $a = 12$, $G_4 = -\frac{3}{2}$

$-\frac{3}{2} = 12 . r^{n-1}$

$-\frac{3}{2} \times \frac{1}{12} = r^{n-1}$

$-\frac{3}{24} = r^3$

$-\frac{1}{8} = r^3$

$r = -\frac{1}{2}$ $\quad S_n = 8\left[1-(-0.5)^n\right]$

$S_\infty = \frac{12}{1+\frac{1}{2}} = 12 \times \frac{2}{3} = 8$

$S_\infty - S_n < 0.001$

$8 - 8\left[1-(-0.5)^n\right] < 0.001$

$(-0.5)^n < \frac{0.001}{8}$

$n > \frac{\log \frac{0.001}{8}}{\log 0.5} = 12.97$

$n = 13$

8 $S_8 = \frac{1}{2} S_\infty$

$$\frac{a(1-r^8)}{1-r} = \frac{1}{2}\,\frac{a}{(1-r)}$$

$$\frac{1-r^8}{1-r} = \frac{1}{2(1-r)}$$

$$(1-r) = 2(1-r)(1-r^8)$$

$$1 = 2(1-r^8)$$

$$1-r^8 = \frac{1}{2}$$

$$\frac{1}{2} = r^8$$

$$r = \sqrt[8]{\frac{1}{2}} = \underline{0\ 917}$$

$$G_{17} = 10 = a \times 0{,}917^{16}$$

$$a = \frac{10}{0\ 917^{16}} = \underline{40}$$

9 $20,\ 110\% \times 20,\ (110\%)^2 \times 20, \ldots$

$$U_n = 20 \times 1\ 1^{n-1}$$

$$U_n > 100$$

$$20 \times 1\ 1^{n-1} > 100$$

$$1.1^{n-1} > 5$$

$$n-1 > \frac{\log 5}{\log 1\ 1}$$

$$n > 1 + \frac{\log 5}{\log 1\ 1} = 17\ 9$$

$$n = 18$$

10 $\overset{0}{6000},\ \overset{1}{1\ 06 \times 6000},\ \overset{2}{1.06^2 \times 6000},\ \overset{10}{1\ 06^{10}} \times 6000 = 10\ \ 745$

Jan 1990 6000

Jan 1991 6000×1.06

Jan 1992 $6000 \times 1\ 06^2$

10 years $\rightarrow$ Jan 2000 $\longrightarrow$ $6000 \times 1\ 06^{10} = 10\ 745$

Jan 90 6000×1.06^{10}

Jan 91 6000×1.06^9

Jan 92 6000×1.06^8

Jan 93 6000×1.06^7

Jan 94 6000×1.06^6

Jan 95 6000×1.06^5

$$\text{Total} = 6000\,(\underbrace{1\ 06^5 + 1\ 06^6 + 1.06^7 + \ldots}_{S_n} + 1.06^{10})$$

$a = 1.06^5,\ r = 1.06,\ n = 6$

$$S_n = \frac{1\ 06\,(1.06^6 - 1)}{0\ 06} = 7\ 39$$

$$\text{Total} = 6000 \times S_n = \underline{44363}$$

16 $S_\infty = 1 + r + r^2 + \dots \quad = k \times (\underbrace{1 - r + r^2 - \dots}_{S_2})$

$a = 1,\ r \qquad\qquad a = 1,\ -r$

$S = \frac{a}{1-r} = k\frac{a}{1+r}$

$k(1-r) = 1 + r$

$k = \frac{1+r}{1-r}$

17 $B = 100\ 000 \qquad 25$ years

$I = 4\%$ per year $\qquad r = 1.04$

0 $\quad 100\ 000$

1 $\quad Br - C$

2 $\quad (Br - C)r - C = Br^2 - Cr - C$

3 $\quad (Br^2 - Cr - C)r - C = Br^3 - Cr^2 - Cr - C$

$\vdots$

25 $\quad Br^{25} - Cr^{24} - Cr^{23} - \dots C$

$0 = Br^{25} - C(r^{24} + r^{23} + \dots r^0)$

$Br^{25} = C(r^{24} + r^{23} + \dots + 1)$

$B = C(\underbrace{\frac{1}{r} + \frac{1}{r^2} + \dots + \frac{1}{r^{25}}}_{S_n})$

$a = \frac{1}{1.04},\ r = \frac{1}{1.04},\ n = 25$

$S_n = \frac{\frac{1}{1.04}\left(1 - \left(\frac{1}{1.04}\right)^{25}\right)}{1 - \frac{1}{1.04}} = 25\,(1 -$

23 $\quad G_1 = 16,\ G_4 = \frac{27}{4}$

$\frac{27}{4} = 16r^3$

$r^3 = \frac{27}{4 \times 16} = \frac{3^3}{4^3}$

$r = \frac{3}{4}$

$S_\infty = \frac{a}{1-r} = \frac{16}{1 - \frac{3}{4}} = 64$

24. I. $a = 1,\ r$

II. $a = 4,\ r' = \frac{1}{4}r$

$\frac{1}{1-r} = \frac{4}{1 - \frac{1}{4}r}$

$1 - \frac{1}{4}r = 4 - 4r$

$4r - \frac{1}{4}r = 3$

$3\frac{3}{4}r = 3$

$\frac{15}{4}r = 3$

$r = \frac{12}{15} = \underline{\underline{\frac{4}{5}}}$

$S_\infty = \frac{1}{1-r}$

$S_\infty = \frac{4}{1 - \frac{1}{4}r}$

$S_\infty = \frac{1}{1 - \frac{4}{5}} = \frac{1}{1/5} = \underline{\underline{5}}$

25 $\quad G_3 = -108$

$G_6 = 32$

i $\quad \frac{G_6}{G_3} = \frac{ar^5}{ar^2} = r^3 = \frac{32}{-108} = -\frac{8}{27}$

$r = -\frac{2}{3}$

ii $\quad G_6 = 32 = a\left(-\frac{2}{3}\right)^5$

$32 = a\left(-\frac{32}{243}\right)$

$a = 32\left(-\frac{243}{32}\right) = \underline{\underline{-243}}$

iii $\quad S_\infty = \frac{a}{1-r} = \frac{-243}{1 + \frac{2}{3}} = -\frac{243}{5/3} = -\frac{729}{5}$

26 $\quad S_\infty = \frac{a}{1-2r} = \frac{3a}{1-r}$

$\frac{1}{1-2r} = \frac{3}{1-r}$

$1 - r = 3 - 6r$

$5r = 2$

$r = \underline{\underline{\frac{2}{5}}}$