MISC EX 8

1 $U_{r+1} = 3U_r - 1,\ U_0 = C$

a

i $C=1$

$1, 2, 5, 14, 41$

$C=2$

$5, 14, 41,$

$C=0$

$-1, -4, -13, -40$

$C=\frac{1}{2}$

$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

$C=2$ $U_1 = \frac{1}{2} + 3b = 5$

$U_2 = \frac{1}{2} + 9b = 14$

$6b = 9,\ b = \frac{9}{6} = \frac{3}{2}$

$U_3 = \frac{1}{2} + 27b = 41$

$27 \cdot \frac{3}{2} = 41 - \frac{1}{2}$

$27 \times 3 = 82 - 1$

2 $U_1 = 0 \quad U_{r+1} = (2+U_r)^2$

$U_2 = 4$

$U_3 = 36$

$U_4 = 38^2$

3 $U_{n+1} = 9 - U_n^2$

a. $U_1 = 3,\ U_2 = 0,\ U_3 = 9,\ U_4 = 9 - 81 = -72$

b $U_1 = U_2$

$a = 9 - a^2$

$a^2 + a - 9 = 0$

$a = \frac{-1 \pm \sqrt{1 - 4(-9)}}{2} = \frac{-1 \pm \sqrt{37}}{2}$

c $U_1 = U_3$

$U_2 = 9 - a^2$

$U_3 = 9 - (9 - a^2)^2 = a$

$9 - (81 - 18a^2 + a^4) - a = 0$

$a^4 - 18a^2 + 81 + a - 9 = 0$

$a^4 - 18a^2 + a + 72 = 0$

4 $a_1 = k$

$a_{n+1} = \frac{1 + a_n}{1 - a_n}$

a $k = \frac{\sqrt{3}}{3}$

$a_2 = \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}}$

$= \frac{9 + 6\sqrt{3} + 3}{6}$

$= \frac{12 + 6\sqrt{3}}{6}$

$= 2 + \sqrt{3}$

$a_3 = \frac{1 + 2 + \sqrt{3}}{1 - (2 + \sqrt{3})}$

$= \frac{3+\sqrt{3}}{-1-\sqrt{3}} = -\frac{3+\sqrt{3}}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$

$= -\frac{3 - 3\sqrt{3} + \sqrt{3} - 3}{-2} = \frac{-2\sqrt{3}}{2}$

$= -\sqrt{3}$

$a_4 = \frac{1-\sqrt{3}}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$

$= \frac{1 - 2\sqrt{3} + 3}{-2} = \frac{4 - 2\sqrt{3}}{-2}$

$= -2 + \sqrt{3}$

$a_5 = \frac{1 - 2 + \sqrt{3}}{1 + 2 - \sqrt{3}} = \frac{-1 + \sqrt{3}}{3 - \sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}}$

$= \frac{-3 - \sqrt{3} + 3\sqrt{3} + 3}{6}$

$= \frac{2\sqrt{3}}{6} = \frac{1}{3}\sqrt{3}$

$a = \frac{1+a}{1-a}$

$a - a^2 = 1 + a$

$a^2 = -1 \Rightarrow$ No roots

5 $U_k = 5k-3$

$d=5,\ U_1 = 2$

6 $U_6 = 23,\ S_{10} = 200,\ U_7 = ?$

$23 = a+5d$

$S_{10} = 200 = \frac{10}{2}(2a+9d)$

$40 = 2a+9d$

$46 = 2a+10d$

$6 = d,\ a = -7$

$U_7 = -7+6\ 6 = 29$

7 $U_5 = 18,\ S_5 = 75$

$18 = a+4d$ $\quad 75 = \frac{5}{2}(2a+4d)$

$15 = a+2d$ $\quad \frac{150}{5} = 2a+4d$

$3 = 2d$

$\underline{\underline{d = \frac{3}{2}}},\ \underline{\underline{a = 12}}$

8 $a = 6,\ U_5 = 12 = 6+4d$

$S_n = 90$ $\quad 6 = 4d$

$d = \frac{6}{4} = \frac{3}{2}$

$90 = \frac{n}{2}\left(12+(n-1)\frac{3}{2}\right)$

$180 = n\left(12+\frac{3}{2}n-\frac{3}{2}\right)$

$180 = 12n+\frac{3}{2}n^2-\frac{3}{2}n$

$3n^2+24n-3n = 360$

$3n^2+21n-360 = 0$

$n^2+7n-120 = 0$

$(n-8)(n+15) = 0$

$n = \underline{\underline{8}}$

9 $U_9 = 22,\ S_4 = 49,\ U_n = 46$

$22 = a+8d$ $\quad 49 = 2(2a+3d)$ $\quad 46 = 10+(n-1)\frac{3}{2}$

$22 = a+12$ $\quad 49 = 4a+6d$ $\quad 36 = \frac{3}{2}(n-1)$

$a = \underline{\underline{10}}$ $\quad 88 = 4a+32d$ $\quad \frac{72}{3}+1 = n$

$39 = 26d$

$d = \frac{39}{26} = \frac{3}{2}$ $\quad 24+1 = n$

$n = \underline{\underline{25}}$

10 $U_1 = 61,\ U_2 = 57 \Rightarrow d = -4$

$S_n = n = \frac{n}{2}\left(122+(n-1)(-4)\right)$

$2n = n(122-4n+4)$

$2n = n(126-4n)$

$2n = 126n-4n^2$

$4n^2-124n = 0$

$n^2-31n = 0$

$n(n-31) = 0$

$\cancel{n=0},\ \underline{\underline{n=31}}$

11. $S_{10} = 400 = 5(2a+9d)$

$S_{20}-S_{10} = 1000$

$S_{20}-400 = 1000$

$S_{20} = 1400 = 10(2a+19d)$

$140 = 2a+19d$

$80 = 2a+9d$

$60 = 10d$

$\underline{\underline{d=6}},\ a = \frac{80-54}{2} = \frac{26}{2} = \underline{\underline{13}}$

12 $a = 7,\ U_n = 84,$ $\quad U_{3n} = 245$

$U_n = 84 = 7+(n-1)d$ $\quad 245 = 7+(3n-1)d$

$84-7 = (n-1)d$ $\quad 238 = (3n-1)d$

$77 = (n-1)d$

$d = \frac{77}{n-1} = \frac{238}{3n-1}$

$\frac{11}{n-1} = \frac{34}{3n-1}$

$11(3n-1) = 34(n-1)$

$33n-11 = 34n-34$

$n = 34-11 = \underline{23}$

13 $U_1 = 19\,000,\ d = 2500$

$U_7 = 19000+6(2500)$

$= 34000$

$S_7 = \frac{7}{2}\left(38\,000+6(2500)\right)$

$= 185500$

$\text{Total} = 185500+3(34000)$

$= 287\,500$