Miscellaneous Exercise 10

1 $f(x) = a - b\cos x$, $0 \leqslant x \leqslant 360$, $a > b$ ⊕

$f(x)$ max $= 10$

$f(x)$ min $= -2$

i $10 = a + b \quad (\cos x = -1)$

$-2 = a - b \quad (\cos x = 1)$

$8 = 2a$

$a = 4,\ b = 6$

ii $f(x) = 0 = 4 - 6\cos x$

$6\cos x = 4$

$\cos x = \frac{4}{6} = \frac{2}{3}$

$x = 48.2°,\ 311.8°$

2 i $2\tan^2\theta \sin^2\theta = 1$

$2\dfrac{\sin^2\theta}{\cos^2\theta} \times \sin^2\theta = 1$

$2\sin^4\theta = \cos^2\theta$

$2\sin^4\theta = 1 - \sin^2\theta$

$2\sin^4\theta + \sin^2\theta - 1 = 0$

ii $2A^2 + A - 1 = 0 \quad (A = \sin^2\theta)$

$(2A - 1)(A + 1) = 0$

$2A = 1 \qquad A = -1$

$A = \frac{1}{2} \qquad \sin^2\theta = -1$

$\sin^2\theta = \frac{1}{2} \qquad \theta = \emptyset$

$\sin\theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{2}\sqrt{2}$

$\theta = 45, 135, 225, 315$

3 $y = \cos\frac{1}{2}\theta$

cross the θ axis @ $y = 0$

$0 = \cos\frac{1}{2}\theta$

$\frac{1}{2}\theta = 90, 270 \pm k\,360$

$\theta = 180, 540 \pm k\,720$

$= 180, -180$

cros the y-axis @ $\theta = 0$

$y = \cos 0 = 1$

4. i $3\sin x \tan x = 8$

$\dfrac{3\sin^2 x}{\cos x} = 8$

$3\sin^2 x = 8\cos x$

$3(1 - \cos^2 x) = 8\cos x$

$3\cos^2 x + 8\cos x - 3 = 0$

ii $3A^2 + 8A - 3 = 0 \qquad A = \cos x$

$(3A - 1)(A + 3) = 0$

$A = \frac{1}{3} \qquad A = -3$

$\cos x = \frac{1}{3} \qquad$ (no roots)

$x = \pm 70.5 \pm k.360 \qquad \therefore x = 70.5, 289.5$

$= 70.5, 289.5$

5 $\dfrac{\sin x}{1 - \sin x} - \dfrac{\sin x}{1 + \sin x} \equiv 2\tan^2 x$

$\dfrac{\sin x(1 + \sin x) - \sin x(1 - \sin x)}{1 - \sin^2 x} \equiv 2\tan^2 x$

$\dfrac{2\sin^2 x}{\cos^2 x} \equiv 2\tan^2 x$

$2\tan^2 x \equiv 2\tan^2 x$

6 i $2\tan^2\theta \cos\theta = 3$

$2\dfrac{\sin^2\theta}{\cos^2\theta} \cdot \cos\theta = 3$

$2\sin^2\theta = 3\cos\theta$

$2(1 - \cos^2\theta) = 3\cos\theta$

$2\cos^2\theta + 3\cos\theta - 2 = 0$

ii $2A^2 + 3A - 2 = 0 \quad (A = \cos\theta)$

$(2A - 1)(A + 2) = 0$

$A = \frac{1}{2} \qquad A = -2$

$\cos\theta = \frac{1}{2} \qquad \cos\theta = -2$ (no roots)

$\theta = 60, 300$

7 i $\frac{\sin x \tan x}{1-\cos x} \equiv 1 + \frac{1}{\cos x}$

$\frac{\sin x \tan x}{1-\cos x} \frac{(1+\cos x)}{(1+\cos x)} \equiv 1 + \frac{1}{\cos x}$

$\frac{\sin x \tan x (1+\cos x)}{\sin^2 x} \equiv 1 + \frac{1}{\cos x}$

$\frac{\sin^2 x (1+\cos x)}{\cos x \sin^2 x} \equiv 1 + \frac{1}{\cos x}$

$\frac{1+\cos x}{\cos x} \equiv 1 + \frac{1}{\cos x}$

ii $\frac{\sin x \tan x}{1-\cos x} + 2 = 0, \quad 0 \leq x \leq 360$

$1 + \frac{1}{\cos x} + 2 = 0$

$\frac{1}{\cos x} + 3 = 0$

$\cos x = -\frac{1}{3}$

$x = 109.5, 250.5$

8 $3\tan(2x+15) = 4 \quad 0 \leq x \leq 180$

$\tan(2x+15) = \frac{4}{3}$

$2x+15 = 53.13 \pm k\,180$

$2x = 38.13 \pm k\,180$

$x = 19.07 \pm k\,90$

$= 19.07, 109.07$ //

9 a $\sin A = 0.2$, $\cos A$ ⊖ $\Rightarrow$ Q2

$A = 168.5$

b $\tan A = -0.5$, $\sin A$ ⊖ $\Rightarrow$ Q4

$A = 333.4$

c. $\cos A = \sin A$ both ⊖ $\Rightarrow$ Q3

$A = 225$

d $\sin A = -0.2275$, $A > 360$

$A = 553.1$ //

10 a $\frac{1}{\sin\theta} - \sin\theta \equiv \frac{\cos\theta}{\tan\theta}$

$\frac{1-\sin^2\theta}{\sin\theta} \equiv \cos\theta \cdot \frac{\cos\theta}{\sin\theta}$

$\frac{\cos^2\theta}{\sin\theta} \equiv \frac{\cos^2\theta}{\sin\theta}$

b. $\frac{1-\sin\theta}{\cos\theta} \equiv \frac{\cos\theta}{1+\sin\theta}$

$\frac{(1-\sin\theta)}{\cos\theta} \frac{(1+\sin\theta)}{(1+\sin\theta)} \equiv \frac{\cos\theta}{1+\sin\theta}$

$\frac{\cos^2\theta}{\cos\theta(1+\sin\theta)} \equiv \frac{\cos\theta}{1+\sin\theta}$

$\frac{\cos\theta}{1+\sin\theta} \equiv \frac{\cos\theta}{1+\sin\theta}$

c. $\frac{1}{\tan\theta} + \tan\theta \equiv \frac{1}{\sin\theta\cos\theta}$

$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \equiv \frac{1}{\sin\theta\cos\theta}$

$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} \equiv \frac{1}{\sin\theta\cos\theta}$

$\frac{1}{\sin\theta\cos\theta} \equiv \frac{1}{\sin\theta\cos\theta}$

d $\frac{1-2\sin^2\theta}{\cos\theta+\sin\theta} \equiv \cos\theta - \sin\theta$

$\frac{1-\sin^2\theta-\sin^2\theta}{\cos\theta+\sin\theta} \equiv \cos\theta - \sin\theta$

$\frac{\cos^2\theta - \sin^2\theta}{\cos\theta+\sin\theta} \equiv \cos\theta - \sin\theta$

$\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\cos\theta+\sin\theta} \equiv \cos\theta - \sin\theta$

11a $y = 1 + \cos 2x$

$y_{max} = 2$

$\cos 2x = 1$

$2x = 0, 360$

$x = 180$ //

$y_{min} = 0$

$\cos 2x = -1$

$2x = 180$

$x = 90$

b $y = 5 - 4\sin(x+30)$

$y_{max} = 5 + 4 = 9$

$\sin(x+30) = -1$

$x + 30 = 270$

$x = 240$

$y_{min} = 5 - 4 = 1$

$\sin(x+30) = 1$

$x + 30 = 90$

$x = \underline{\underline{60}}$

c $y = 29 - 20\sin(3x - 45)^\circ$

$y_{max} = 29 + 20 = 49$

$\sin(3x - 45) = -1$

$3x - 45 = -90 \pm k\,360$

$3x = 45 - 90 \pm k\,360$

$x = -15 \pm k\,120 = \underline{\underline{105}}$

$y_{min} = 29 - 20 = 9$

$\sin(3x - 45) = 1$

$3x - 45 = 90$

$3x = 135$

$x = \underline{\underline{45}}$

d. $y = 8 - 3\cos^2 x$

$y_{max} = 8$

$\cos^2 x = 0$

$x = 90$

$y_{min} = 5$

$\cos^2 x = 1$

$\cos x = \pm 1$

$x = 0, 180, 360$

e $y = \dfrac{12}{3 + \cos x}$

$y_{max} = \dfrac{12}{3-1} = 6$

$\cos x = -1$

$x = 180$

$y_{min} = \dfrac{12}{3+1} = 3$

$\cos x = 1$

$x = 360$

f $y = \dfrac{60}{1 + \sin^2(2x - 15)}$

$y_{max} = \dfrac{60}{1} = 60$

$\sin^2(2x - 15) = 0$

$2x - 15 = 0, 180$

$2x = 15, 195$

$x = 75, 975$

$y_{min} = \dfrac{60}{1+1} = 30$

$\sin^2(2x - 15) = 1$

$\sin(2x - 15) = \pm 1$

$2x - 15 = 90, 270$

$2x = 105, 285$

$x = 525, 1425$ //

12a $\sin\theta = \tan\theta$

$\sin\theta - \tan\theta = 0$

$\sin\theta - \dfrac{\sin\theta}{\cos\theta} = 0$

$\sin\theta\left(1 - \dfrac{1}{\cos\theta}\right) = 0$

$\sin\theta = 0$ or $\dfrac{1}{\cos\theta} = 1$

$\theta = 0, 180, 360$

b $2 - 2\cos^2\theta = \sin\theta$

$2 - 2(1 - \sin^2\theta) = \sin\theta$

$2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0$ or $\sin\theta = \frac{1}{2}$

$\theta = 0, 180, 360$ $\theta = 30, 150$

$\therefore \theta = 0, 30, 150, 180, 360$

c $\tan^2\theta - 2\tan\theta - 1 = 0$

$\tan\theta = \dfrac{+2 \pm \sqrt{4 - 4(1)(-1)}}{2} = \dfrac{2 \pm 2\sqrt{2}}{2}$

$= 1 + \sqrt{2}$ or $1 - \sqrt{2}$

$\theta = 67.5, 247.5, 337.5, 157.5$ //

d. $\sin 2\theta - \sqrt{3}\cos 2\theta = 0$ $\div \cos 2\theta$

$\tan 2\theta - \sqrt{3} = 0$

$\tan 2\theta = \sqrt{3}$

$2\theta = 60^\circ \pm k.180$

$\theta = 30^\circ \pm k.90$

$\therefore \theta = 30, 120, 210, 300$ //

13 i. $4\sin^2 x + 8\cos x - 7 = 0, \quad 0 \leq x \leq 360$

$4(1-\cos^2 x) + 8\cos x - 7 = 0$

$4\cos^2 x - 8\cos x + 7 - 4 = 0$

$4\cos^2 x - 8\cos x + 3 = 0$

$(2\cos x - 1)(2\cos x - 3) = 0$

$\cos x = \frac{1}{2}$ or $\cos x = \frac{3}{2}$ (not possible)

$x = 60, 300$ //

ii $4\sin^2\left(\frac{1}{2}\theta\right) + 8\cos\left(\frac{1}{2}\theta\right) - 7 = 0$

$\cos\left(\frac{1}{2}\theta\right) = \frac{1}{2}$

$\frac{1}{2}\theta = 60 \pm k.360, -60 \pm k.360$

$\theta = 120 \pm k.720, -120 \pm k.720$

$= 120$ //

19 $\frac{13\sin^2\theta}{2+\cos\theta} + \cos\theta = 2, \quad 0 \leq \theta \leq 180$

$\frac{13(1-\cos^2\theta)}{2+\cos\theta} + \cos\theta = 2$

$13(1-\cos^2\theta) + \cos\theta(2+\cos\theta) = 2(2+\cos\theta)$

$13 - 13\cos^2\theta + 2\cos\theta + \cos^2\theta = 4 + 2\cos\theta$

$0 = 12\cos^2\theta - 9$

$3(4\cos^2\theta - 3) = 0$

$3(2\cos\theta + \sqrt{3})(2\cos\theta - \sqrt{3}) = 0$

$2\cos\theta = -\sqrt{3}$ or $2\cos\theta = \sqrt{3}$

$\cos\theta = -\frac{1}{2}\sqrt{3}$ $\qquad$ $\cos\theta = \frac{1}{2}\sqrt{3}$

$\theta = 150, -150$ $\qquad$ $\theta = 30, -30$

$\therefore \theta = 30, 150$ //

14.a $y_{max} = 6m = a + b$

$y_{min} - 3.6m = a - b$

$9.6 = 2a$

$a = 4.8, \; b = 1.2$

$y = 4.8 + 1.2\sin\left(\frac{t}{24} \times 360\right)$

$y = 4.8 + 1.2\sin 15t$

20 $2\cos^2\theta = 3\sin\theta \quad 0 \leqslant \theta \leqslant 360$

$2(1-\sin^2\theta) = 3\sin\theta$

$2 - 2\sin^2\theta = 3\sin\theta$

$2\sin^2\theta + 3\sin\theta - 2 = 0$

$(2x-1)(x+2) = 0$

$x = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

$\theta = 30, 150$ /

ii $2\cos^2(n\theta) = 3\sin(n\theta), \quad n \in I$

$2(1-\sin^2 n\theta) - 3\sin n\theta = 0$

$2\sin^2 n\theta + 3\sin n\theta - 2 = 0$

$(2\sin n\theta - 1)(\sin n\theta + 2) = 0$

$\sin n\theta = \frac{1}{2}$

$n\theta = 30, \; \theta = 10$

$n = 3$ //

$3\theta = 30, 150 \pm k.360$

$\theta = 10, 50 \pm k.120$

$\therefore \theta = 10, 50, 130, 170, 250, 290$