

# Miscellaneous Exercise 18

1. Sector Area =  $\frac{\theta}{2} r^2$

Triangle Area =  $\frac{1}{2} r \cdot Ox \sin \theta$

=  $\frac{1}{2} r \cdot r \cos \theta \cdot \sin \theta$

Shaded Area

=  $\frac{\theta}{2} r^2 - \frac{1}{2} r^2 \cos \theta \sin \theta$

=  $\frac{1}{2} r^2 (\theta - \sin \theta \cos \theta)$  shown!

$r = 12, \theta = \frac{1}{6} \pi$

Perimeter =  $XB + AX + AB$

=  $(12 - 12 \cos \frac{\pi}{6}) + 12 \sin \frac{\pi}{6} + \frac{\pi}{6} \cdot 12$

=  $12 - 12(\frac{1}{2}\sqrt{3}) + 12(\frac{1}{2}) + 2\pi$

=  $12 - 6\sqrt{3} + 6 + 2\pi = 18 - 6\sqrt{3} + 2\pi$

$\cos \theta = \frac{Ox}{r}$

$Ox = r \cos \theta$

$\sin \theta = \frac{Ax}{r}$

$Ax = r \sin \theta$

5.  $\cos \theta = \frac{OC}{r}$

$OC = r \cos \theta$

i  $AC = r - OC = r - r \cos \theta$

=  $r(1 - \cos \theta) = 4(\frac{1}{2}) = 2$

ii  $BC = r \sin \theta$

$BD = r \sin \theta - DC = r \sin \theta - r(1 - \cos \theta)$

=  $r \sin \theta + r \cos \theta - r = 4(\sin \frac{\pi}{3} + \cos \frac{\pi}{3} - 1)$

=  $4(\frac{1}{2}\sqrt{3} + \frac{1}{2} - 1) = 2\sqrt{3} - 2$

$AB = \theta \times r = \frac{\pi}{3} \times 4 = \frac{4\pi}{3}$

$AD = \frac{\pi}{2} \times r(1 - \cos \theta) = \frac{\pi}{2} \times 4 \times (1 - \cos \frac{\pi}{3})$

=  $2\pi(1 - \frac{1}{2}) = \pi$

Perimeter =  $BD + AD + AB$

=  $2\sqrt{3} - 2 + \pi + \frac{4\pi}{3} = 2\sqrt{3} - 2 + \frac{7\pi}{3}$

2.  $\theta = \tan^{-1}(\frac{12}{5}) =$

$S = \theta \times r = \tan^{-1}(\frac{12}{5}) \times 5$

= 5.88 cm

i. Perimeter =  $5.88 + 12 + 8 = 25.88$  cm

ii. Area = Area triangle - Area Sector

=  $\frac{1}{2}(12)(5) - \frac{\tan^{-1}(\frac{12}{5})}{2} \times 5^2$

= 15.3 cm<sup>2</sup>

6. b.  $\cos x(1 + \sin x) = 0$

$\cos x = 0$  or  $\sin x = -1$

$x = \frac{\pi}{2}, -\frac{\pi}{2} \pm k2\pi$       $x = -\frac{\pi}{2}, \frac{3\pi}{2} \pm k.2\pi$

$\therefore x = \pm \frac{\pi}{2}$

3. i  $a = 6, b = 2, c = 3$

ii  $y = 6 \sin 2x + 3, y = 0$

$0 = 6 \sin 2x + 3$

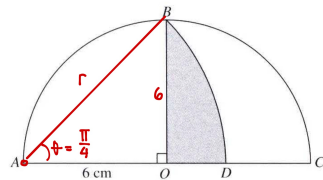
$6 \sin 2x = -3$

$\sin 2x = -\frac{1}{2}$

$2x = -\frac{\pi}{6}, \frac{7\pi}{6} \pm k.2\pi$

$x = -\frac{\pi}{12}, \frac{7\pi}{12} \pm k \frac{12}{2}\pi$

=  $\frac{7}{12}\pi$



4. i  $r = 6\sqrt{2}$

$\theta = \frac{\pi}{4}$

$S = BD = \frac{\pi}{4} \times 6\sqrt{2} = \frac{3}{2}\sqrt{2}\pi$

ii. Area triangle =  $\frac{1}{2} \times 6 \times 6 = 18$

Sector Area =  $\frac{\pi}{4} \times (6\sqrt{2})^2 = \frac{\pi}{8} \times 72 = 9\pi$

Shaded Area

=  $9\pi - 18 \approx 10.3$  cm<sup>2</sup>

c.  $(1 - \tan x) \sin x = 0$

$\tan x = 1$  or  $\sin x = 0$

$x = \frac{\pi}{4} \pm k\pi$       $x = 0, \pi \pm k.2\pi$

$\therefore x = -\frac{3\pi}{4}, 0, \frac{\pi}{4}, \pi$

e.  $\cos(\frac{3\pi}{4} - x) = 0.832$

$\frac{3\pi}{4} - x = 0.588, -0.588 \pm k.2\pi$

$x = 1.768, 2.944$

f.  $\tan(3x - 17) = 3$

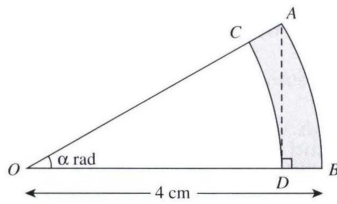
$3x - 17 = 1.249 \pm k.\pi$

$3x = 18.249 \pm k.\pi$

$x = 6.08 \pm k \frac{\pi}{3}$

= 2.94, 1.89, 0.85, -0.2, -1.25, -2.29

9.



$$AB = 4\alpha$$

$$AD = 4 \sin \alpha$$

$$BD = 4 - OD = 4 - 4 \cos \alpha$$

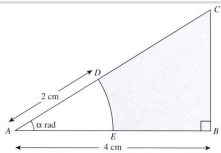
$$CD = \alpha \cdot 4 \cos \alpha$$

$$\begin{aligned} \text{i. Perimeter} &= 2(4 - 4 \cos \alpha) + 4\alpha + 4\alpha \cos \alpha \\ &= 8 - 8 \cos \alpha + 4\alpha + 4\alpha \cos \alpha \end{aligned}$$

$$\text{ii. } \alpha = \frac{\pi}{6}$$

$$\begin{aligned} \text{Area of } ABDC &= \frac{\alpha}{2}(4)^2 - \left(\frac{\alpha}{2}\right)(4 \cos \alpha)^2 \\ &= \frac{\pi}{6} \times (8) - \frac{\pi}{6} \times (8) \cos^2 \left(\frac{\pi}{6}\right) \\ &= \frac{4}{3}\pi - \frac{4}{3}\pi \left(\frac{1}{2}\sqrt{3}\right)^2 = \frac{4\pi}{3} \left(1 - \frac{3}{4}\right) = \frac{4\pi}{3} \left(\frac{1}{4}\right) = \frac{1}{3}\pi \end{aligned}$$

10.



$$\tan \alpha = \frac{BC}{AB} \quad \cos \alpha = \frac{AB}{AC}$$

$$BC = AB \tan \alpha \quad AC = \frac{4}{\cos \alpha}$$

$$= 4 \tan \alpha$$

$$CD = AC - 2 = \frac{4}{\cos \alpha} - 2$$

$$DE = 2\alpha$$

$$\begin{aligned} \text{i. Area} &= \frac{1}{2} \times 4 \times 4 \tan \alpha - \frac{\alpha}{2} (2)^2 \\ &= 8 \tan \alpha - 2\alpha \end{aligned}$$

$$\begin{aligned} \text{ii. Perimeter} &= DE + BE + BC + CD \\ &= 2\alpha + 2 + 4 \tan \alpha + \frac{4}{\cos \alpha} - 2 \\ &= 2\alpha + 4 \tan \alpha + \frac{4}{\cos \alpha} // \end{aligned}$$

$$14. \text{ a. } 2 \sin^{-1} x - 4$$

$$\text{Domain: } -1 \leq x \leq 1$$

$$\text{Range: } -\pi - 4 \leq y \leq \pi - 4$$

$$\text{b. } 2 \sin^{-1} (x-4)$$

$$\text{Domain: } -1 \leq x-4 \leq 1$$

$$\text{Range: } 3 \leq x \leq 5$$

$$15. 3 \sin^2 \theta + 4 \cos \theta = 4, \quad 0 \leq \theta \leq 2\pi, \quad 2 \text{ dp}$$

$$3(1 - \cos^2 \theta) + 4 \cos \theta = 4$$

$$3 \cos^2 \theta - 4 \cos \theta + 1 = 0$$

$$(3 \cos \theta - 1)(\cos \theta - 1) = 0$$

$$3 \cos \theta = 1 \quad \text{or} \quad \cos \theta = 1$$

$$\cos \theta = \frac{1}{3}$$

$$\theta = 0, 2\pi$$

$$\theta = 1.23, -1.23 \pm k \cdot 2\pi$$

$$= 1.23, 5.05$$

$$\therefore \theta = 0, 1.23, 5.05, 2\pi,$$

$$16. -2\pi \leq \theta \leq 2\pi$$

$$\text{a. } 2 \cos^2 \theta + \sin^2 \theta = 0$$

$$2 \cos^2 \theta + (1 - \cos^2 \theta) = 0$$

$$\cos^2 \theta + 1 = 0$$

$$\theta = \text{no roots.}$$

$$\text{b. } 2 \cos^2 \theta + \sin^2 \theta = 1$$

$$2 \cos^2 \theta + (1 - \cos^2 \theta) = 1$$

$$\cos^2 \theta = 0$$

$$\theta = \pm \frac{\pi}{2} \pm k \cdot 2\pi$$

$$\therefore \theta = -\frac{3}{2}\pi, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3}{2}\pi$$

$$\text{c. } 2 \cos^2 \theta + \sin^2 \theta = 2$$

$$2 \cos^2 \theta + (1 - \cos^2 \theta) = 2$$

$$\cos^2 \theta - 1 = 0$$

$$\cos \theta = \pm 1$$

$$\theta = -2\pi, -\pi, 0, \pi, 2\pi //$$