CHAPTER TEST

1. $f(x) = 3x-5$
$g(x) = x^2$
$h(x) = \frac{2}{x}$

a. $fg = f(x^2) = 3x^2-5$ [1]

b $hfg = hf(x^2) = h(3x^2-5) = \frac{2}{3x^2-5}$ [2]

c $fh^{-1} = ?$
$y = \frac{2}{x} \rightarrow x = \frac{2}{y}$
$h^{-1}(x) = \frac{2}{x}$ [1]
$f(\frac{2}{x}) = 3(\frac{2}{x}) - 5 = \frac{6}{x} - 5$ [2]

d $f(x) = 3x-5$
$f^{-1}(x) = \frac{x+5}{3}$ [1]

$f^{-1}g = f^{-1}(x^2)$
$= \frac{x^2+5}{3}$ [1]

p174 # 13, 14
p182 #9, 10

3. $f(x) = \frac{2x+1}{x}$
$fg(x) = x-2$
$\frac{2g+1}{g} = x-2$ [1]
$2g+1 = g(x-2)$ [1]
$2g - (x-2)g = -1$
$g(2-x+2) = -1$
$g = -\frac{1}{4-x} = \frac{1}{x-4}$ [1]

4 $g(x) = x+1$
$fg(x) = x^2+3x+2$
$f(x+1) = x^2+3x+2$ [1]
$a = x+1$ $f(a) = (a-1)^2+3(a-1)+2$ [1]
$x = a-1$ $= (a^2-2a+1)+3a-3+2$
$= a^2+a$ [1]
$f(x) = x^2+x$

2 i $-x^2+6x-5$
[1] $= -(x^2-6x)-5$
[1] $= -[(x-3)^2-9]-5$
[1] $= -(x-3)^2+4$
$a = -1, b = -3, c = 4$

p185 # 12
p182# 13-15

p 181 #7

$(x+1)^2 + (x+1) = x^2+2x+1+x+1$
$= x^2+3x+2$

ii $f(x) = -x^2+6x-5$
$x \geq m$ one-one function
[1] $m = 3$

Ex 10 A
#3,4,5

iii If $m = 5 \rightarrow$ domain $x \geq 5$
$f(5) = -25+30-5$
$= 0$
[1] Range $f(x) \leq 0 \rightarrow$ Domain for $f^{-1}(x)$
$y = -(x-3)^2+4$
[1] $4-y = (x-3)^2$
[1] $x-3 = +\sqrt{4-y}$
[1] $x = 3+\sqrt{4-y}$
$f^{-1}(x) = 3+\sqrt{4-x}$
[1] Domain : $x \leq 0$
Range : $f^{-1}(x) \geq 5$

Ex 10A #2
Misc Ex 10 # 11

5a $\sin(-20) = \sin(340) = \sin(200)$ $0 \leq x \leq 360$
b $\cos(-120) = \cos(240) = \cos(120)$ $0 \leq x \leq 360$
c $\tan(1200) = \tan(120) = \tan(-60)$ $-180 \leq x \leq 180$

6 a. $y = 10+2\cos x$
$y_{min} = 10+2(-1) = 8$ [1]
$\cos x = -1$
$x = 180$ [1]
$y_{max} = 10+2(1) = 12$ [1]
$\cos x = 1$
$x = 360$ "0" is not positive [1]

b $y = \frac{24}{10 - 2\sin(2x+30)}$

$y_{max} = \frac{24}{10-2(1)} = 3$ [1]

$\sin(2x+30) = 1$

$2x+30 = 90$ [1]

$2x = 60$

$x = 30$ [1]

$y_{min} = \frac{24}{10-2(-1)} = 2$ [1]

$\sin(2x+30) = -1$

$2x+30 = -90 \pm k\,360$ [1]

$2x = -120 \pm k.360$

$x = -60 \pm k\,180$

$x = 120$ [1]

Ex 10 D #2, Misc Ex 10 # 9

7 a $\sin^2 A = 1 - \cos^2 A = 1 - \frac{9}{25}$ [1]

$= \frac{16}{25}$

$\sin A = -\frac{4}{5}$

$\tan A = \frac{\sin A}{\cos A} = \frac{-4/5}{3/5} = -\frac{4}{3}$ [1]

b $A = -53.1, 126.9$

8a $\sin 2\theta = -0.643, \quad 0 \leq \theta \leq 180$ Ex 10 C #6,7 Ex 18 D #1-6

$2\theta = -40 + k\,360$ $\qquad 2\theta = 220 \pm k.360$

$\theta = -20 \pm k.180$ $\qquad \theta = 110 \pm k.180$

$\therefore \theta = 110, 160$

b. $\cos\frac{3}{2}\theta = -0.643, \quad -\pi \leq \theta \leq \pi$

$\frac{3}{2}\theta = 2.27 \pm k\,2\pi$ $\qquad \frac{3}{2}\theta = -2.27 \pm k.2\pi$

$\theta = 1.51 \pm k\frac{4}{3}\pi$ $\qquad \theta = -1.51 \pm k.\frac{4\pi}{3}$

$= 1.51, -2.68$ $\qquad = -1.51, 2.68$

g $\sin^2\theta = 2\cos\theta, \quad -\pi \leq \theta \leq \pi$ Ex 10D #4,5 Ex 18 D #7

$1 - \cos^2\theta = 2\cos\theta$ [1]

$\cos^2\theta + 2\cos\theta - 1 = 0$

$\cos\theta = \frac{-2 \pm \sqrt{4 - 4(-1)}}{2}$ [1]

$= \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$ [1]

$\cos\theta = -1 - \sqrt{2}$ $\qquad \cos\theta = -1 + \sqrt{2}$

No roots $\qquad \theta = \pm 1.14 \pm k.2\pi$

$= 1.14, -1.14$ [2]

10a $1 - 3\sin^2\theta \equiv 3\cos^2\theta - 2$

$1 - 3(1 - \cos^2\theta) \equiv 3\cos^2\theta - 2$ [1]

$1 - 3 + 3\cos^2\theta \equiv 3\cos^2\theta - 2$ [1]

$3\cos^2\theta - 2 \equiv 3\cos^2\theta - 2$

b $\frac{\tan\theta}{1 - \tan^2\theta} \equiv \frac{\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

$\frac{\sin\theta}{\cos\theta} \div \left(1 - \frac{\sin^2\theta}{\cos^2\theta}\right) \equiv \frac{\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ [1]

$\frac{\sin\theta}{\cos\theta} \div \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta} \equiv \frac{\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ [1]

$\frac{\sin\theta}{\cos\theta} \times \frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta} \equiv \frac{\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ [1]

$\frac{\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} \equiv \frac{\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$