MATHEMATICS REVISION
SEMESTER 1 EXAMINATION
ACADEMIC YEAR 2019-2020

Mathematics-9709/2019-May-June/9709_s19_qp_11

1 The term independent of x in the expansion of $\left(2x + \frac{k}{x}\right)^6$, where k is a constant, is 540.

(i) Find the value of k. $\frac{3}{2}$

(ii) For this value of k, find the coefficient of x^2 in the expansion. 540

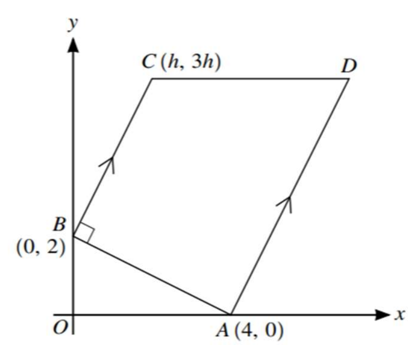
2 The line $4y = x + c$, where c is a constant, is a tangent to curve $y^2 = x + 3$ at point P on the curve.

(i) Find the value of c. 7

(ii) Find the coordinates of P. (1,2)

3 A sector of a circle of radius r cm has an area of A cm^2. Express the perimeter of the sector in terms of r and A.

4 The diagram shows a trapezium $ABCD$ in which the coordinates of A, B and C are $(4, 0)$, $(0, 2)$ and $(h, 3h)$ respectively. The lines BC and AD are parallel angle $ABC = 90°$ and CD is parallel to the x-axis.



(i) Find, by calculation, the value of h. 2

(ii) Hence find the coordinates of D.

5 The function f is defined by $f(x) = -2x^2 + 12x - 3$ for $x \in \mathbb{R}$.

(i) Express $-2x^2 + 12x - 3$ in the form $-2(x+a)^2 + b$, where a and b are constants.

(ii) State the greatest value of $f(x)$.

The function g is defined by $g(x) = 2x + 5$ for $x \in \mathbb{R}$.

(iii) Find the values of x for which $gf(x) + 1 = 0$. $x = 0, 6$

6 **(i)** Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)^2 \equiv \frac{1-\sin x}{1+\sin x}$.

(ii) Hence solve the equation $\left(\frac{1}{\cos 2x} - \tan 2x\right)^2 = \frac{1}{3}$ for $0 \leq x \leq \pi$.

7 **(a)** The third and fourth terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression.

(b) Two schemes are proposed for increasing the amount of household waste that is recycled each week.

Scheme A is to increase the amount of waste recycled each month by 0.16 tonnes.

Scheme B is to increase the amount of waste recycled each month by 6% of the amount recycled in the previous month.

The proposal is to operate the scheme for a period of 24 months. The amount recycled in the first month is 2.5 tonnes.

For each scheme, find the total amount of waste that would be recycled over the 24-month period.

1. $\left(2x+\frac{k}{x}\right)^6$

(i) Term independent of x:

$\binom{6}{3}(2x)^3\left(\frac{k}{x}\right)^3 = 20(8)k^3 = 540$

$k^3 = \frac{540}{20 \times 8} = \frac{27}{8}$

$k = \sqrt[3]{\frac{27}{8}} = \underline{\underline{\frac{3}{2}}}$

(ii) $\binom{6}{2}(2x)^4\left(\frac{3/2}{x}\right)^2 = 15(16)\frac{9}{4}x^2 = 540$

Coeff of x^2: $\underline{\underline{540}}$

2 (i) $4y = x + c$

$y^2 = x + 3$

$y^2 - 4y = 3 - c$

$y^2 - 4y - 3 + c = 0$ $\begin{cases} a = 1 \\ b = -4 \\ c = -3 + c \end{cases}$

tangent $\rightarrow D = 0$

$(-4)^2 - 4(-3+c) = 0$

$16 + 12 - 4c = 0$

$28 = 4c$

$c = 7$

(ii) $y^2 - 4y - 3 + 7 = 0$

$y^2 - 4y + 4 = 0$

$(y-2)^2 = 0$

$y = 2, x = 4y - 7$

$= 1$

$P(1, 2)$

3. $A = \frac{\theta}{2}r^2 \Rightarrow \theta = \frac{2A}{r^2}$

$P = \theta r + 2r$

$= \frac{2A}{r^2} \times r + 2r = \underline{\underline{\frac{2A}{r} + 2r}}$

4. $A(4,0)$

$B(0,2)$

(i) $m_{AB} = -\frac{1}{2}$

$m_{\perp} = 2$

line BC:

$y - 2 = 2(x - 0)$

$y = 2x + 2$

$C(h, 3h)$:

$3h = 2h + 2$

$h = 2 \rightarrow C(2,6)$

(ii) line AD:

$y - 0 = 2(x - 4)$

$y = 2x - 8$

line CD: $y = 6$ } intercept at D

$6 = 2x - 8$

$14 = 2x$

$x = 7, y = 6$ $D(7,6)$

5. $f(x) = -2x^2 + 12x - 3$

(i) $-2(x^2 - 6x) - 3$

$-2\left[(x-3)^2 - 9\right] - 3$

$\underline{\underline{-2(x-3)^2 + 15}}$

(ii). Greatest value of $f(x)$ is $\underline{\underline{15}}$

$g(x) = 2x + 5$

$gf(x) + 1 = 0$

$g(-2x^2 + 12x - 3) + 1 = 0$

$2(-2x^2 + 12x - 3) + 5 + 1 = 0$

$-4x^2 + 24x = 0$

$-4x(x - 6) = 0$

$x = 0$ or $x = 6$

6 (i) $\left(\frac{1}{\cos x} - \tan x\right)^2 \equiv \frac{1-\sin x}{1+\sin x}$

$\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2 \equiv \frac{1-\sin x}{1+\sin x}$

$\left(\frac{1-\sin x}{\cos x}\right)^2 \equiv \frac{1-\sin x}{1+\sin x}$

$\frac{(1-\sin x)(1-\sin x)}{\cos^2 x} \equiv \frac{1-\sin x}{1+\sin x}$

$\frac{(1-\sin x)(1-\sin x)}{1-\sin^2 x} \equiv \frac{1-\sin x}{1+\sin x}$

$\frac{(1-\sin x)\cancel{(1-\sin x)}}{(1+\sin x)\cancel{(1-\sin x)}} \equiv \frac{1-\sin x}{1+\sin x}$

(ii) $\frac{1-\sin 2x}{1+\sin 2x} = \frac{1}{3}$

$1+\sin 2x = 3 - 3\sin 2x$

$4\sin 2x = 2$

$\sin 2x = \frac{1}{2}$

① $2x = \frac{\pi}{6} \pm k.2\pi$

$x = \frac{\pi}{12} \pm k.\pi$

② $2x = \frac{5\pi}{6} \pm k.2\pi$

$x = \frac{5\pi}{12} \pm k.\pi$

$\therefore x = \frac{\pi}{12}, \frac{5\pi}{12}$

(0.262 , 1.31)

7a. $U_3 = 48 = a.r^2$

$U_4 = 32 = ar^3$

$\frac{U_4}{U_3} = \frac{32}{48} = \frac{ar^3}{ar^2} = r$

$r = \frac{2}{3}$

$48 = a.\left(\frac{2}{3}\right)^2$

$48 = \frac{4}{9}a$

$a = 48 \times \frac{9}{4} = 108$

$S_\infty = \frac{108}{1-\frac{2}{3}} = 324$

b. Scheme A : $a = 2.5$ tonnes

$d = 0.16$

$S_{24} = \frac{24}{2}(2 \times 2.5 + 23 \times 0.16) = \underline{104.16}$ tonnes

Scheme B : $a = 2.5$ tonnes

$r = 1 + 6\% = 1.06$

$S_{24} = \frac{2.5(1.06^{24}-1)}{1.06-1} = \underline{127.04}$ tonnes

MATHEMATICS REVISION 2
SEMESTER 1 EXAMINATION
ACADEMIC YEAR 2019-2020

Mathematics-9709/2018-May-June/9709_s18_qp_11.pdf

1 **(i)** Find the first three terms in the expansion, in ascending powers of x, of $(1-2x)^5$.

(ii) Given that the coefficient of x^2 in the expansion of $(1+ax+2x^2)(1-2x)^5$ is 12, find the value of the constant a.

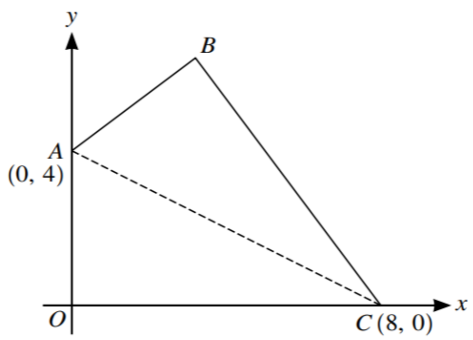
2 **(i)** Prove the identity $(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)\equiv\sin^3\theta+\cos^3\theta$.

(ii) Hence solve the equation $(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)=3\cos^3\theta$ for $0°\le\theta\le 360°$.

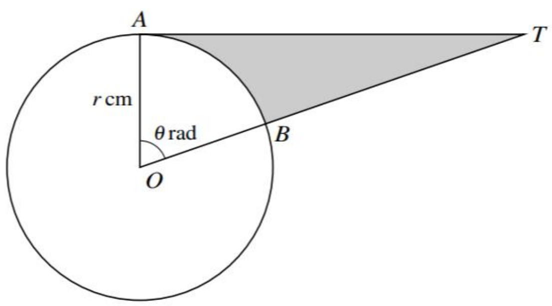
3 The diagram shows a kite $OABC$ in which AC is the line of symmetry. The coordinates of A and C are $(0,4)$ and $(8,0)$ respectively and O is the origin.

(i) Find the equation of AC and OB.

(ii) Find, by calculation, the coordinates of B.



4 The diagram shows a circle with centre O and radius r cm. The points A and B lie on the circle and AT is a tangent to the circle. Angle $AOB=\theta$ radians and OBT is a straight line.



(i) Express the area of the shaded region in terms of r and θ.

(ii) In the case where $r=3$ and $\theta=1.2$, find the perimeter of the shaded region.

5 **(a)** A geometric progression has a second term of 12 and a sum to infinity of 54.

Find the possible values of the first term of the progression.

(b) The nth term of a progression is $p+qn$, where p and q are constants, and S_n is the sum of the first n terms.

(i) Find an expression, in terms of p, q and n, for S_n.

(ii) Given that $S_4=40$ and $S_6=72$, find the values of p and q.

1.(I) $(1-2x)^5$

$= \binom{5}{0}(-2x)^0 + \binom{5}{1}(-2x) + \binom{5}{2}(-2x)^2$

$= 1 - 10x + 40x^2$ //

(II) $(1+ax+2x^2)(1-2x)^5$

$= (1+ax+2x^2)(1-10x+40x^2)$

Coeff of x^2

$40x^2 - 10ax^2 + 2x^2 = 12x^2$

$42 - 10a = 12$

$30 = 10a$

$a = 3$

2. $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) = \sin^3\theta + \cos^3\theta$

$\sin\theta - \sin^2\theta\cos\theta + \cos\theta - \sin\theta\cos^2\theta$

$\sin\theta - (1-\cos^2\theta)\cos\theta + \cos\theta - \sin\theta(1-\sin^2\theta)$

$\sin\theta - \cos\theta + \cos^3\theta + \cos\theta - \sin\theta + \sin^3\theta$

$\sin^3\theta + \cos^3\theta \equiv$

$\sin^3\theta + \cos^3\theta = 3\cos^3\theta$

$\sin^3\theta = 2\cos^3\theta$

$\tan^3\theta = 2$

$\tan\theta = \sqrt[3]{2}$

$\theta = 51.56, 231.56$ //

3. $A(0,4)$

$C(8,0)$

(I) $m_{AC} = -\frac{1}{2}$

Line AC:

$y = -\frac{1}{2}(x-8)$

$y = -\frac{1}{2}x + 4$ //

Line OB:

$m = 2$, $(0,0)$

$y = 2x$ //

(II) Intersection of AC & OB:

$y = -\frac{1}{2}x + 4$

$y = 2x$

$2x = -\frac{1}{2}x + 4$

$4x = -x + 8$

$5x = 8$

$x = \frac{8}{5}$, $y = \frac{16}{5}$ $\left(\frac{8}{5}, \frac{16}{5}\right)$ D

OD = DB

$B = \left(\frac{16}{5}, \frac{32}{5}\right)$

4(I). $\tan\theta = \frac{AT}{r}$

$AT = r\tan\theta$

Triangle Area $= \frac{1}{2} \times r \times r\tan\theta = \frac{1}{2}r^2\tan\theta$

Sector Area $= \frac{\theta}{2} \times r^2$

Shaded region $= \frac{1}{2}r^2\tan\theta - \frac{\theta}{2}r^2$ //

(II) $r = 3$, $\theta = 1.2$

Perimeter = AT + AB + BT

$AB = \theta r = 1.2 \times 3 = 3.6$

$\cos\theta = \frac{r}{OT}$

$OT = \frac{r}{\cos\theta} = \frac{3}{\cos 1.2}$

$BT = OT - r = \frac{3}{\cos 1.2} - 3 = 5.28$

$AT = 3\tan 1.2 = 7.72$

Perimeter $= 7.72 + 3.6 + 5.28 = 16.6$ cm

5. $G_2 = a.r = 12 \Rightarrow a = \frac{12}{r}$

$S_\infty = 54 = \frac{a}{1-r}$

$54 = \frac{12}{r(1-r)}$

$54r(1-r) = 12$

$54r^2 - 54r + 12 = 0$

$9r^2 - 9r + 2 = 0$

$(3r-2)(3r-1) = 0$

$r = \frac{2}{3}$ or $\frac{1}{3}$

① $r = \frac{2}{3}$, $a = \frac{12}{2/3} = 18$

② $r = \frac{1}{3}$, $a = \frac{12}{1/3} = 36$

6. $U_n = p + qn$

$U_1 = p + q$

$U_2 = p + 2q$

$U_3 = p + 3q$

(I) $S_n = \frac{n}{2}(U_1 + U_n)$

$S_n = \frac{n}{2}(p + q + p + nq)$

$= \frac{n}{2}(2p + q + nq)$

(II) $S_4 = 40 = \frac{4}{2}(2p + q + 4q)$

$40 = 2(2p + 5q)$

$20 = 2p + 5q$

$4 = 2q$

$q = 2$ //

$S_6 = 72 = \frac{6}{2}(2p + q + 6q)$

$72 = 3(2p + 7q)$

$24 = 2p + 7q$

$20 = 2p + 5(2)$

$10 = 2p$

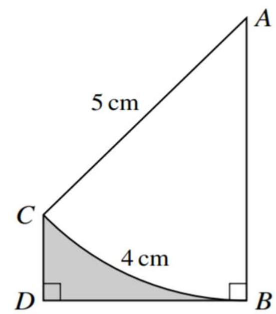
$p = 5$ //

MATHEMATICS REVISION 3
SEMESTER 1 EXAMINATION
ACADEMIC YEAR 2019-2020

Mathematics-9709/2018-Oct-Nov/9709_w18_qp_13.pdf

1 Find the coefficient of $\frac{1}{x^3}$ in the expansion of $\left(x - \frac{2}{x}\right)^7$.

2 The diagram shows and arc BC of a circle with centre A and radius 5 cm. The length of the arc BC is 4 cm. The point D is such that the line BD is perpendicular to BA and DC is parallel to BA.



(i) Find angle BAC in radians.

(ii) Find the area of the shaded region BDC.

3 Two points A and B have coordinates $(-1, 1)$ and $(3, 4)$ respectively. The line BC is perpendicular to AB and intersects the x-axis at C.

(i) Find the equation of BC and the x-coordinate of C.

(ii) Find the distance AC, giving your answer correct to 3 decimal places.

(iii) Find the midpoint of AB.

4 In an arithmetic progression the first term is a and the common difference is 3. The nth term is 94 and the sum of the first n terms is 1420. Find n and a.

5 **(i)** Show that $\frac{\tan\theta+1}{1+\cos\theta} + \frac{\tan\theta-1}{1-\cos\theta} \equiv \frac{2(\tan\theta-\cos\theta)}{\sin^2\theta}$.

(ii) Hence, showing all necessary working, solve the equation

$$\frac{\tan\theta + 1}{1 + \cos\theta} + \frac{\tan\theta - 1}{1 - \cos\theta} = 0$$

for $0° < \theta < 90°$.

6 A curve has equation $y = 2x^2 - 3x + 1$ and a line has equation $y = kx + k^2$, where k is a constant.

(i) Show that, for all values of k, the curve and the line meet.

(ii) State the value of k for which the line is a tangent to the curve and find the coordinates of the point where the line touches the curve.

1. $\left(x-\frac{2}{x}\right)^7 = \binom{7}{0}x^7 + \binom{7}{1}x^6\left(-\frac{2}{x}\right) + \binom{7}{2}x^5\left(-\frac{2}{x}\right)^2$

Coeff of $\frac{1}{x^3}$:

$\binom{7}{5}x^2\left(-\frac{2}{x}\right)^5 = 21\,(-32)\left(\frac{1}{x^3}\right) = \underline{-672}$

2. $4 = \theta \times 5$

$\theta = \frac{4}{5}$ rad

$\frac{BD}{5} = \sin\theta$

$BD = 5\sin 0.8 = 3.59$ cm

$DC = 5 - 5\cos 0.8 = 1.52$ cm

Trapezium Area $= (DC+5) \times \frac{BD}{2}$

$= (1.52+5) \times \frac{3.59}{2} = 11.7$ cm^2

Shaded Area $= 11.7 - \frac{0.8}{2} \times 25 = \underline{1.7}$ cm^2

3. $A(-1,1)$

$B(3,4)$

$m_{AB} = \frac{3}{4}$

$m_{BC} = -\frac{4}{3}$

Line BC :

$y - 4 = -\frac{4}{3}(x-3)$

$3y - 12 = -4x + 12$

(i) $4x + 3y = 24$

Intersect at x-axis = $y = 0$

$4x + 0 = 24$

$x = 6$ $\quad \underline{C(6,0)}$

(ii) $A(-1,1)$ $C(6,0)$

$d = \sqrt{7^2 + 1} = \sqrt{50} =$

(iii) Mid Point of AB $\left(1, \frac{5}{2}\right)$

4. $d = 3$, $U_n = 94$, $S_n = 1420$

$94 = a + 3(n-1)$

$94 = a + 3n - 3$

$97 = a + 3n$

$a = 97 - 3n$

$1420 = \frac{n}{2}(a + 94)$

$2840 = n(97 - 3n + 94)$

$2840 = n(191 - 3n)$

$3n^2 - 191n + 2840 = 0$

$n_{1,2} = \frac{191 \pm \sqrt{191^2 - 4(3)(2840)}}{6}$

$= \frac{191 \pm 49}{6} = \underline{40}$ or $\frac{71}{3}$

$a = 97 - 3n$

$= 97 - 3(40)$

$= 97 - 120 = \underline{-23}$

5i $\frac{\tan\theta + 1}{1 + \cos\theta} + \frac{\tan\theta - 1}{1 - \cos\theta} \equiv \frac{2(\tan\theta - \cos\theta)}{\sin^2\theta}$

$\frac{(\tan\theta + 1)(1 - \cos\theta) + (\tan - 1)(1 + \cos\theta)}{1 - \cos^2\theta} \equiv$

$\frac{\tan\theta - \sin\theta + 1 - \cos\theta + \tan\theta + \sin\theta - 1 - \cos\theta}{\sin^2\theta}$

$\frac{2\tan\theta - 2\cos\theta}{\sin^2\theta} = \frac{2(\tan\theta - \cos\theta)}{\sin^2\theta}$

(ii) $\frac{2(\tan\theta - \cos\theta)}{\sin^2\theta} = 0$

$2\tan\theta = 2\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \cos\theta$

$\sin\theta = \cos^2\theta$

$\sin\theta = 1 - \sin^2\theta$

$\sin^2\theta + \sin\theta - 1 = 0$

$\sin\theta = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

$= -1.62$ or 0.62

$\theta = 38.2^\circ$

6. $\left.\begin{array}{l} y = 2x^2 - 3x + 1 \\ y = kx + k^2 \end{array}\right\}$

$2x^2 - 3x + 1 = kx + k^2$

$2x^2 - 3x - kx + 1 - k^2 = 0$

$2x^2 - (3+k)x + (1-k^2) = 0$

(i) $D = (3+k)^2 - 4(2)(1-k^2) \geqslant 0$

$= 9 + 6k + k^2 - 8 + 8k^2 \geqslant 0$

$9k^2 + 6k + 1 \geqslant 0$

$(3k+1)^2 \geqslant 0$ shown!

(ii) tangent : $D = 0$

$(3k+1)^2 = 0$

$k = -\frac{1}{3}$

$2x^2 - \frac{8}{3}x + \frac{8}{9} = 0$

$9x^2 - 12x + 4 = 0$

$(3x-2)(3x-2) = 0$

$x = \frac{2}{3}$

$y = -\frac{1}{3}x + \left(-\frac{1}{3}\right)^2$

$= -\frac{2}{9} + \left(\frac{1}{9}\right) = -\frac{1}{9}$//

Coordinates $\left(\frac{2}{3}, -\frac{1}{9}\right)$