

EXERCISE 12 D

$$1. N = \frac{20000}{1+0.2t} = 20000 (1+0.2t)^{-1}$$

$$\frac{dN}{dt} = -20000 \cdot 0.2 (1+0.2t)^{-2}$$

$$t=5, \frac{dN}{dt} = -4000 (1+1)^{-2}$$

$$= -\frac{4000}{4} = -\underline{1000}$$

$$\textcircled{2} y = \frac{9}{2-x} = 9(2-x)^{-1} \rightarrow \text{After CH7}$$

$$\frac{dy}{dx} = 9(2-x)^{-2} = 0$$

$$\frac{9}{(2-x)^2} = 0$$

\therefore No stationary points

$$3. \frac{dl}{dt} = 1.2 \text{ cm/s}$$

$$l = 10 \text{ cm}$$

$$a. P = 4l$$

$$\frac{dP}{dt} = \frac{dP}{dl} \times \frac{dl}{dt}$$

$$= 4 \times 1.2 = 4.8 \text{ cm/s}$$

$$b. A = l^2$$

$$\frac{dA}{dl} = 2l = 20 \text{ cm}$$

$$\frac{dA}{dt} = \frac{dA}{dl} \times \frac{dl}{dt} = 20 \times 1.2 = 24 \text{ cm}^2/\text{s}$$

$$4. \frac{dl}{dt} = 0.5 \text{ mm/s}$$

$$l = 40 \text{ mm}$$

$$a. \frac{dA}{dt} = ?$$

$$A = 6 \cdot l^2$$

$$\frac{dA}{dl} = 12l = 12 \cdot 40 \text{ mm} = 480 \text{ mm}$$

$$\frac{dA}{dt} = \frac{dA}{dl} \times \frac{dl}{dt} = 480 \text{ mm} \times 0.5 \text{ mm/s}$$

$$= 240 \text{ mm}^2/\text{s}$$

$$b. \frac{dV}{dt} = ?$$

$$V = l^3$$

$$\frac{dV}{dl} = 3l^2 = 3 \cdot 40^2 = 4800 \text{ mm}^2$$

$$\frac{dV}{dt} = \frac{dV}{dl} \times \frac{dl}{dt} = 4800 \text{ mm}^2 \times 0.5 \text{ mm/s}$$

$$= 2400 \text{ mm}^3/\text{s}$$

$$5. \frac{dr}{dt} = 3 \text{ mm/s}$$

$$r = 50 \text{ mm}$$

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r = 2\pi \cdot 50 \text{ mm}$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$$= 100\pi \times 3 \text{ mm/s}$$

$$= 300\pi \text{ mm}^2/\text{s} = 942.5 \text{ mm}^2/\text{s}$$

$$6. V = 1.5 \times 1.2 \times l = 1.8 l$$

$$\frac{dV}{dt} = 0.45 \text{ m}^3/\text{minute}$$

$$\frac{dV}{dl} = 1.8$$

$$\frac{dV}{dt} = \frac{dV}{dl} \times \frac{dl}{dt}$$

$$0.45 = 1.8 \times \frac{dl}{dt}$$

$$\frac{dl}{dt} = \frac{0.45}{1.8} = 0.25 \text{ m/min}$$

$$7. \frac{dV}{dt} = 0.6 \text{ m}^3\text{s}^{-1}$$

$$r = 2.5 \text{ m}, \frac{dr}{dt} = ?$$

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$0.6 \text{ m}^3/\text{s} = 4\pi r^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{0.6 \text{ m}^3/\text{s}}{4\pi (2.5)^2 \text{ m}^2} = 0.0076 \text{ m/s}$$

$$8. \frac{dV}{dt} = 0.25 \text{ m}^3/\text{s}$$

$$V = 10 \text{ m}^3, \frac{dr}{dt} = ?$$

$$V = \frac{4}{3} \pi r^3$$

$$10 = \frac{4}{3} \pi r^3$$

$$r = \sqrt[3]{\frac{30}{4\pi}}$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$0.25 \text{ m}^3/\text{s} = 4\pi \sqrt[3]{\left(\frac{30}{4\pi}\right)^2} \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = 0.0111 \text{ m/s}$$

$$9. \frac{dV}{dt} = 0.2 \text{ cm}^3/\text{s}$$

$$h = 12 \text{ cm}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$\frac{h}{30} = \frac{2r}{20}$$

$$V = \frac{1}{3} \pi \left(\frac{1}{5}h\right)^2 h$$

$$2r = \frac{20}{30}h$$

$$= \frac{1}{27} \pi h^3$$

$$r = \frac{1}{3}h$$

$$\frac{dV}{dh} = \frac{1}{9} \pi h^2 = \frac{1}{9} \pi (12)^2 \text{ cm}^2$$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$0.2 \text{ cm}^3/\text{s} = \frac{144\pi}{9} \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.2 \text{ cm}^3/\text{s} \times \frac{9}{144\pi} \text{ cm}^2$$

$$= 0.0040 \text{ cm/s}$$