EXERCISE 6D

1 d $f(x) = 2x^3 - 3x^2 + x$

$f'(x) = 6x^2 - 6x + 1$

e $f(x) = 2x^2(1 - 3x^2) = 2x^2 - 6x^4$

$f'(x) = 4x - 24x^3$

2 $f'(-2)$

d $f(x) = 2 - x$

$f'(x) = -1$

e $f(x) = x^2(1 + x) = x^2 + x^3$

$f'(x) = 2x + 3x^2$

$f'(-2) = -4 + 12 = 8$

3 d $f(x) = x^3 - 3x^2 + 2x$

$f'(x) = 3x^2 - 6x + 2 = -1$

$3x^2 - 6x + 3 = 0$

$3(x^2 - 2x + 1) = 0$

$3(x-1)^2 = 0$

$x = 1$

e $f(x) = x(1 + x)^2 = x(1 + 2x + x^2)$

$f(x) = x^3 + 2x^2 + x$

$f'(x) = 3x^2 + 4x + 1 = 0$

$(3x + 1)(x + 1) = 0$

$x = -\frac{1}{3}, x = -1$

4 g $f(x) = \frac{(x+1)(x+2)}{x} = \frac{x^2 + 3x + 2}{x} = x + 3 + \frac{2}{x}$

$f'(x) = 1 - \frac{2}{x^2}$

h $f(x) = \left(\frac{\sqrt{x} + x}{\sqrt{x}}\right)^2 = \left(1 + x^{\frac{1}{2}}\right)^2 = 1 + 2x^{\frac{1}{2}} + x$

$f'(x) = x^{-\frac{1}{2}} + 1 = 1 + \frac{1}{\sqrt{x}}$

5 $y = x^3 + x$, $x = -1$, $y = -2$ $(-1, -2)$

$m = y' = 3x^2 + 1 = 4$

$y + 2 = 4(x + 1)$

$y = 4x + 2$

6 $y = 4x - x^3$

$m = y' = 4 - 3x^2 = 1$

$3 = 3x^2$

$x = -1$, or $x = 1$

@ $x = -1$, $y = -4 + 1 = -3$ $(-1, -3)$

$y + 3 = x + 1$

$y = x - 2$

@ $x = 1$, $y = 4 - 1 = 3$ $(1, 3)$

$y - 3 = x - 1$

$y = x + 2$

7 $y = \sqrt{x} = x^{\frac{1}{2}}$ at $(4,2)$

$m = y' = \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2}\frac{1}{\sqrt{4}} = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x-4)$

$y = \frac{1}{4}x + 1$

8 $y = \frac{1}{x} = x^{-1}$

$m = y' = -\frac{1}{x^2}$ at $(2, \frac{1}{2})$

$= -\frac{1}{4}$

$y - \frac{1}{2} = -\frac{1}{4}(x-2)$

$y = -\frac{1}{4}x + 1$

9 $y = x + x^{-1}$

$m = y' = 1 - \frac{1}{x^2}$ at $(1,2)$

$m = 1 - 1 = 0 \rightarrow$ line // x axis

Normal eq$_n$ $x = 1$

10 $y = x^2 - 2x$

$m = 2x - 2$

at $(0,0)$ $m = -2$

$y = -2x$

$y = x^3 - 3x^2 - 2x$

$m = 3x^2 - 6x - 2$

at $(0,0)$ $m = -2$

11 $y = x^3 - 3x^2 - 2x - 6$ crosses y-axis $x = 0$

$m = y' = 3x^2 - 6x - 2$ $y = -6$

$m = -2$ $(0,-6)$

$y + 6 = -2x$

$y = -2x - 6$

12 $y = x(x-a)(x+a) = x(x^2 - a^2) = x^3 - a^2x$

$y' = m = 3x^2 - a^2$ $(0,0)$, $(a,0)$, $(-a,0)$

at $(0,0)$ $m = -a^2$

$y = -a^2x$

at $(a,0)$ $m = 2a^2$

$y = 2a^2(x-a) = 2a^2x - 2a^3$

at $(-a,0)$ $m = 2a^2$

$y = 2a^2(x+a) = 2a^2x + 2a^3$

13 $y = x^2$

$m = y' = 2x$

$y = x^2$ & $y = x + 2$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

$x = 2, y = 4, m = 2x = 4$

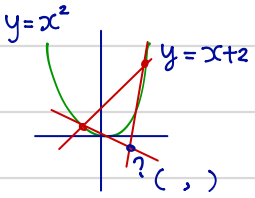
$y - 4 = 4(x-2)$

$y = 4x - 4$

$x = -1, y = 1, m = -2$

$y - 1 = -2(x+1)$

$y = -2x - 1$



$4x - 4 = -2x - 1$

$6x = 3$

$x = \frac{1}{2}$, $y = -2$ $(\frac{1}{2}, -2)$

15 $y = \sqrt[3]{x^2} = x^{\frac{2}{3}}$

$m = y' = \frac{2}{3}x^{-\frac{1}{3}}$

at $(8,4)$ $m = \frac{2}{3}\frac{1}{\sqrt[3]{8}} = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$

tangent eq$_n$.

$y - 4 = \frac{1}{3}(x-8)$

$y = \frac{1}{3}x - \frac{8}{3} + 4$

$y = \frac{1}{3}x + \frac{4}{3}$

normal eq$_n$

$y - 4 = -3(x-8)$

$y = -3x + 28$

16 $y = \frac{1}{x^2} = x^{-2}$

$m = y' = -2x^{-3} = -\frac{2}{x^3}$

at $\left(\frac{1}{2}, 4\right)$ $m = -\frac{2}{1/8} = -16$

$y - 4 = -16\left(x - \frac{1}{2}\right)$

$y = -16x + 12$ meets x & y axes

x axis $y = 0 = -16x + 12$

$16x = 12$

$x = \frac{12}{16} = \frac{3}{4}$ $\left(\frac{3}{4}, 0\right)$

y axis

$x = 0$, $y = 12$ $(0, 12)$