MISCELLANEOUS EXERCISE 6 pgg

1. $y = 5x^2 - 7x + 4$, $(2, 10)$

$y' = 10x - 7$

$m_t = 10(2) - 7 = 13$

$y - 10 = 13(x - 2)$

$y = 13x - 26 + 10$

$y = 13x - 16$ //

2 $f(x) = x^3 + 5x^2 - x - 4$

a $f'(x) = 3x^2 + 10x - 1$

$f'(-2) = 12 - 20 - 1 = -9$

b $f'(a) = 56$

$3a^2 + 10a - 1 = 56$

$3a^2 + 10a - 57 = 0$

$(3a + 19)(a - 3) = 0$

$a = -\frac{19}{3}$, 3

3 $y = x^4 - 4x^3$

$m = y' = 4x^3 - 12x^2$

at $x = \frac{1}{2}$, $m = 4\left(\frac{1}{8}\right) - 12\left(\frac{1}{4}\right)$

$= \frac{1}{2} - 3 = -\frac{5}{2}$

$m_n = \frac{2}{5}$ $\quad y = \frac{1}{16} - 4\left(\frac{1}{8}\right)$

$= \frac{1}{16} - \frac{1}{2} = -\frac{7}{16}$

$y + \frac{7}{16} = \frac{2}{5}\left(x - \frac{1}{2}\right)$

$y = \frac{2}{5}x - \frac{1}{5} - \frac{7}{16}$

$80y = 32x - 51$

4. $y = \frac{1}{x}$, $x = p \rightarrow \left(p, \frac{1}{p}\right)$

$y' = -x^{-2} = -\frac{1}{x^2}$

$m_t = -\frac{1}{p^2}$

$p^2 \times \left(y - \frac{1}{p}\right) = -\frac{1}{p^2}(x - p) \times p^2$

$p^2 y - p = -(x - p)$

$p^2 y - p = -x + p$

$\begin{cases} p^2 y + x = 2p \text{ (shown!)} \\ gy + x = -6 \end{cases}$

$p^2 = 9 \qquad 2p = -6$

$p = \pm 3 \qquad p = -3$

$\therefore \left(-3, -\frac{1}{3}\right)$ //

5. $y = 6\sqrt{x} = 6 \cdot x^{\frac{1}{2}}$ $\quad (4, 12)$

$y' = 3x^{-\frac{1}{2}} = \frac{3}{\sqrt{x}}$

$m_t = \frac{3}{\sqrt{4}} = \frac{3}{2}$

$y - 12 = \frac{3}{2}(x - 4)$

$y = \frac{3}{2}x - 6 + 12$

$y = \frac{3}{2}x + 6 \rightarrow$ meets axes

$A(x, 0) \quad 0 = \frac{3}{2}x + 6$

$-6 = \frac{3}{2}x$

$x = -4 \qquad A(-4, 0)$

$B(0, y) \quad y = 6 \qquad B(0, 6)$

Distance AB

$= \sqrt{(-4)^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$

$k = 2$,

6 $y = 2x^3 - 5x^2 + 9x - 1$

$m = y' = 6x^2 - 10x + 9 = 13$

$6x^2 - 10x - 4 = 0$

$3x^2 - 5x - 2 = 0$

$(3x + 1)(x - 2) = 0$

$x = -\frac{1}{3}$, 2

$x = -\frac{1}{3}$, $y = 2\left(-\frac{1}{3}\right)^3 - 5\left(-\frac{1}{3}\right)^2 + 9\left(-\frac{1}{3}\right) - 1$

$y = -\frac{2}{27} - \frac{5}{9} - 3 - 1 = -4\frac{17}{27} \qquad \left(-\frac{1}{3}, -4\frac{17}{27}\right)$

$x = 2$, $y = 2(2)^3 - 5(2)^2 + 9(2) - 1$

$y = 16 - 20 + 18 - 1 = 13 \qquad (2, 13)$

7. $y=(2x-1)(3x+5)$ $(1,8)$

$y=6x^2+7x-5$

$y'=12x+7$

$m_t=12(1)+7=19$

$m_n=-\frac{1}{19}$

$y-8=-\frac{1}{19}(x-1)$

$19y-152=-x+1$

$x+19y-153=0$ //

9. $y=2x^2-5x+14$ $(1,11)$

$y'=4x-5$

$m_t=4(1)-5=-1$

$m_n=1$

$y-11=x-1$

$\begin{cases} y=x+10 \rightarrow \text{Normal line eq} \\ y=2x^2-5x+14 \end{cases}$

$x+10=2x^2-5x+14$

$2x^2-6x+4=0$

$x^2-3x+2=0$

$(x-2)(x-1)=0$

$x=2$ or $x=1$

$y=2+10=12$ $y=1+10=11$

$(2,12)$ $(1,11)$

10. $y=x^2+k \rightarrow y'=2x$

$y=6x-7 \rightarrow m_t=6$

$y'=m_t=6=2x$

$x=3$

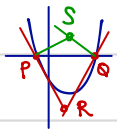
$y=6(3)-7=11$ $\}(3,11)$

$y=x^2+k$

$11=3^2+k$

$k=2$

8



$y=x^2-3x-4$ $m=y'=2x-3$

at x axis $y=0=x^2-3x-4$

$(x-4)(x+1)=0$

$x=4$, $x=-1$ $P(4,0)$ & $Q(-1,0)$

at P, $m=2(4)-3=5$, $m_n=-\frac{1}{5}$

tangent. normal

$y=5(x-4)$ $y=-\frac{1}{5}(x-4)$

$y=5x-20$ $y=-\frac{1}{5}x+\frac{4}{5}$

at Q, $m=2(-1)-3=-5$, $m_n=\frac{1}{5}$

tangent normal

$y=-5(x+1)$ $y=\frac{1}{5}(x+1)$

$y=-5x-5$ $y=\frac{1}{5}x+\frac{1}{5}$

Tangent & tangent Normal & Normal

$5x-20=-5x-5$ $-\frac{1}{5}x+\frac{4}{5}=\frac{1}{5}x+\frac{1}{5}$

$10x=15$ $\frac{3}{5}=\frac{2}{5}x$

$x=15$, $y=-125$ $3=2x$,

$R(1.5,-125)$ $x=15$, $y=05$

$S(15,05)$

$RS=125+05=13$ units

11. $y = 3 + 4x - x^2$

i $m_t = y' = 4 - 2x$

at $(3,6)$ $m_t = 4 - 2(3) = -2$

$m_n = \frac{1}{2}$

normal eqn:

$y - 6 = \frac{1}{2}(x - 3)$

$2y - 12 = x - 3$

$2y = x + 9$ //

ii meets x axis at:

$y = 0$, $x = -9$ $(-9, 0)$

meets y axis at:

$x = 0$, $y = 4.5$ $(0, 4.5)$

Mid Point of AB: $(-4.5, 2.25)$

iii $y = 3 + 4x - x^2$

$2y = x + 9$

$x = 2y - 9$

$y = 3 + 4(2y - 9) - (2y - 9)^2$

$y = 3 + 8y - 36 - (4y^2 - 36y + 81)$

$4y^2 - 43y + 114 = 0$

$(4y - 19)(y - 6) = 0$

$y = \frac{19}{4}$ $y = 6$

$x = 2\left(\frac{19}{4}\right) - 9 = \frac{1}{2}$ $x = 2(6) - 9 = 3$

$\left(\frac{1}{2}, \frac{19}{4}\right)$ $(3,6)$

12. $y = 5x^2 - 12x + 1$

normal eqn: $x + 18y + c = 0$

$18y = -x - c$

$y = -\frac{1}{18}x - \frac{1}{18}c \Rightarrow m_n = -\frac{1}{18}$

$m_t = 18$

$y' = m_t = 10x - 12 = 18$

$10x = 30$

$x = 3$, $y = 5(3^2) - 12(3) + 1$

$= 45 - 36 + 1 = 10$

$(3, 10)$, $m_n = -\frac{1}{18}$

$y - 10 = -\frac{1}{18}(x - 3)$

$18y - 180 = -x + 3$

$x + 18y - 183 = 0$

$c = -183$

13. $y = x^m$ $y = x^n$

$y' = mx^{m-1}$ $y' = nx^{n-1}$

at $(1,1)$: $y' = m$ $y' = n$

$m = -\frac{1}{n}$ or $m \times n = -1$ //

14. $y=\sqrt{x}$, $x=\frac{1}{4}$, $y=\frac{1}{2}$ $\left(\frac{1}{4},\frac{1}{2}\right)$

$m_t = y' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}\left[\left(\frac{1}{2}\right)^2\right]^{-\frac{1}{2}}$

$= \frac{1}{2}\cdot 2 = 1$

tangent eqn:

$y-\frac{1}{2} = x-\frac{1}{4}$

$y = x+\frac{1}{4}$

$y= x^{-\frac{1}{2}}$, $x=\frac{1}{4}$, $y=2$ $\left(\frac{1}{4},2\right)$

$m_t = y' = -\frac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2}\left[\left(\frac{1}{2}\right)^2\right]^{-\frac{3}{2}}$

$= -\frac{1}{2}\cdot 8 = -4$

tangent eqn:

$y-2 = -4\left(x-\frac{1}{4}\right)$

$y = -4x+3$

$x+\frac{1}{4} = -4x+3$

$5x = \frac{11}{4}$

$x = \frac{11}{20}$, $y = \frac{4}{5}$

$P\left(\frac{11}{20},\frac{4}{5}\right)$

15. $y=\frac{1}{x^2}$, $x=2$, $y=\frac{1}{4}$

$m_t = y' = -2x^{-3} = -2\left(\frac{1}{8}\right) = -\frac{1}{4}$

$m_n = 4$

normal eqn:

$y-\frac{1}{4} = 4(x-2)$

$y = 4x-8+\frac{1}{4}$

$y = 4x-\frac{31}{4}$

$y= x^{-3}$, $x=2$, $y=\frac{1}{8}$

$m_t = -3x^{-4} = -3\left(\frac{1}{16}\right)$

$m_n = \frac{16}{3}$

normal eqn:

$y-\frac{1}{8} = \frac{16}{3}(x-2)$

$y = \frac{16}{3}x-\frac{32}{3}+\frac{1}{8}$

$4x-\frac{31}{4} = \frac{16}{3}x-\frac{32}{3}+\frac{1}{8}$

$\frac{4}{3}x = -\frac{31}{4}+\frac{32}{3}-\frac{1}{8}$

$\frac{4}{3}x = -\frac{63}{8}+10\frac{2}{3}$

$\frac{4}{3}x = -7\frac{7}{8}+10\frac{2}{3}$

$\frac{4}{3}x = \frac{67}{24}$

$x = \frac{67}{24}\times\frac{3}{4} = \frac{67}{32} = 2\frac{3}{32}$

$y = 4\left(\frac{67}{32}\right)-\frac{31}{4} = \frac{67}{8}-\frac{62}{8} = \frac{5}{8}$

$\left(\frac{67}{32},\frac{5}{8}\right)$