Miscellaneous Exercise 12

1. $\frac{d(7x-3)^6}{dx} = 42(7x-3)^5$

2. $\frac{d(3x+8)^{-4}}{dx} = (-4)(3)(3x+8)^{-5}$
$= \frac{-12}{(3x+8)^5}$

3. $\frac{d\,2(x^4+3)^5}{dx} = 10(x^4+3)^4(4x^3)$
$= 40x^3(x^4+3)^4$

4. $y = (x^2-5)^6$ at $(2,1)$
$\frac{dy}{dx} = 6(2x)(x^2-5)^5$, $x=2$
$y' = m_t = 24(4-5)^5 = -24$
$y-1 = -24(x-2)$
$y = -24x + 49$ //

5. $y = \sqrt{x^3+1} = (x^3+1)^{\frac{1}{2}}$
$\frac{dy}{dx} = \frac{1}{2}(x^3+1)^{-\frac{1}{2}}(3x^2)$
$= \frac{3x^2}{2\sqrt{x^3+1}}$ $\Rightarrow x^3+1 > 0$
$x^3 > -1$
$x > -1$

6. $y = \frac{1}{2x-1} + \frac{1}{(2x-1)^2} = (2x-1)^{-1} + (2x-1)^{-2}$
$\frac{dy}{dx} = -\frac{2}{(2x-1)^2} - \frac{4}{(2x-1)^3}$, $x=2$
$\frac{dy}{dx} = -\frac{2}{9} - \frac{4}{27} = -\frac{10}{27}$ //

7. $y = 2 - \frac{18}{2x+3} = 2 - 18(2x+3)^{-1}$
$A(x,0)$ $B(0,y)$
$0 = 2 - \frac{18}{2x+3}$ $y = 2 - \frac{18}{3} = -4$
$\frac{18}{2x+3} = 2$ $B(0,-4)$
$2x+3 = 9$
$2x = 6$
$x = 3$ $A(3,0)$
$y' = 36(2x+3)^{-2} = \frac{36}{(2x+3)^2}$
at $A(3,0)$: $m_t = \frac{36}{81} = \frac{4}{9}$
$m_n = -\frac{9}{4}$
$y = -\frac{9}{4}(x-3)$
$C(0,y)$ $y = -\frac{9}{4}(0-3)$
$= \frac{27}{4}$
$C(0,\frac{27}{4})$ $A(3,0)$
$m_{AC} = -\frac{27/4}{3} = -\frac{9}{4}$
$y = -\frac{9}{4}(x-3)$

(i) $4y = -9x + 27$

(ii) $B(0,-4)$ $C(0,\frac{27}{4})$
$BC = \frac{27}{4} + 4 = 10\frac{3}{4}$ unit

8. $y = \frac{1}{x^2+4} = (x^2+4)^{-1}$

$y' = -\frac{2x}{(x^2+4)^2} = 0$

$x = 0$

$y = \frac{1}{4}$

Stationary point $(0, \frac{1}{4})$ //

9. $y = (2x^2+1)^{\frac{1}{2}}$ $(2,3)$

$y' = \frac{2x}{\sqrt{2x^2+1}}$

$m_t = \frac{4}{3}$, $m_n = -\frac{3}{4}$

$y - 3 = -\frac{3}{4}(x-2)$

$y = -\frac{3}{4}x + \frac{3}{2} + 3$

$y = -\frac{3}{4}x + \frac{9}{2}$ //

10. $\frac{dr}{dt} = 3\text{cm s}^{-1}$ $A = \pi r^2$

$\frac{dA}{dt} = ?$ $r = 9$ cm $\frac{dA}{dr} = 2\pi r = 18\pi$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi r \text{ cm} \times 3 \text{ cm/s}$

$= 169.6 \text{ cm}^2/\text{s}$

12. $y = (4x+3)^{-1}$

$\frac{dy}{dx} = -\frac{4}{(4x+3)^2}$, $x = -1$, $y = -1$

$(-1,-1)$

$m_t = -\frac{4}{1} = -4$

$m_n = \frac{1}{4}$

$y + 1 = \frac{1}{4}(x+1)$

$y = \frac{1}{4}x - \frac{3}{4}$ → normal line eq.

$P(x,0)$: $0 = \frac{1}{4}x - \frac{3}{4}$ $Q(0,y)$: $y = 0 - \frac{3}{4}$

$\frac{3}{4} = \frac{1}{4}x$ $Q(0, \frac{3}{4})$ //

$x = 3$

$P(3,0)$

11. $y = \frac{12}{x^2+3} = 12(x^2+3)^{-1}$

(i) $\frac{dy}{dx} = -12(x^2+3)^{-2}(2x)$

$= -24x(x^2+3)^{-2}$

(ii) $P(1,3)$ $m_t = -24(4)^{-2}$

$= -\frac{24}{16} = -\frac{3}{2}$

$m_n = \frac{2}{3}$ $P(1,3)$

$y - 3 = \frac{2}{3}(x-1)$

$3y = 2x - 2 + 9$

$3y = 2x + 7$

(iii) $\frac{dx}{dt} = 0.012$ units/s

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$= -24x(x^2+3)^{-2} \times 0.012$ units/s

$P(1,3)$: $= -24(4)^{-2} \times 0.012 = -\frac{24}{16} \times 0.012$

$= -\frac{3}{2} \times 0.012 = -0.018$

13. $r = 50$m $A = \pi r^2$

$\frac{dr}{dt} = 3$m/h $\frac{dA}{dr} = 2\pi r = 100\pi$ m

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$= 100\pi \times 3 = 300\pi$ m^2/s

14. $y = 4(3x+1)^{-2}$

$\frac{dy}{dx} = -\frac{24}{(3x+1)^3}$, $x = -1$, $y = 1$

$(-1, 1)$

$m_t = -\frac{24}{-8} = 3$

$y - 1 = 3(x+1)$

$y = 3x + 4$

15. $\frac{dV}{dt} = 50\,cm^3/s$ $\quad r = 10\ cm$

$\frac{dr}{dt} = ?$ $\quad V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$50 = 4\pi r^2 \times \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{50}{4\pi r^2} = \frac{50}{4\pi 100} = \frac{1}{8\pi}\ cm/s$

16. $y = (6x+2)^{\frac{1}{3}}$

$A(1,2)$

$y' = \frac{6}{3}(6x+2)^{-\frac{2}{3}} = 2(6x+2)^{-\frac{2}{3}}$

i $m_t = 2(8)^{-\frac{2}{3}} = \frac{1}{2}$

$m_n = -2$

AB:

$y - 2 = \frac{1}{2}(x-1)$

$y = \frac{1}{2}x + \frac{3}{2}$

AC:

$y - 2 = -2(x-1)$

$y = -2x + 4$

ii B: $(0, y)$

$y = \frac{1}{2}x + \frac{3}{2} = \frac{3}{2}$

$B(0, \frac{3}{2})$

C: $(x, 0)$

$0 = -2x + 4$

$x = 2$ $\quad C(2,0)$

$BC = \sqrt{4 + \frac{9}{4}} = \frac{5}{2}$

iii

$m_{BC} = -\frac{3}{4}$

$y = -\frac{3}{4}(x-2)$

$y = -\frac{3}{4}x + \frac{3}{2}$

$m_{OA} = 2$

$y - 2 = 2(x-1)$

$y = 2x$

E: $2x = -\frac{3}{4}x + \frac{3}{2}$

$\frac{11}{4}x = \frac{3}{2}$

$x = \frac{3}{2} \times \frac{4}{11} = \frac{6}{11}$

$y = \frac{12}{11}$ $\quad E(\frac{6}{11}, \frac{12}{11})$

Mid Point of OA $(\frac{1}{2}, 1)$

$\therefore$ E is not Midpoint of OA

17. $M = kr^3$ $\quad \frac{dM}{dr} = 3kr^2$

$\frac{dr}{dt} = 0.1$ cm/day

$r = 10$cm, $m = 3.2$ kg

$3.2 = k \cdot 10^3$

$k = 3.2 \times 10^{-3}$

$\frac{dM}{dt} = \frac{dM}{dr} \times \frac{dr}{dt}$

$= 3kr^2 \times 0.1$

$= 3 \times 3.2 \times 10^{-3} \times 10^2 \times 0.1$

$= 9.6 \times 10^{-2}$ kg/day

20. $r = 6$cm, $h = x$ cm

$V = \frac{1}{3}\pi x^2 (18 - x) = 6\pi x^2 - \frac{1}{3}\pi x^3$

$\frac{dV}{dt} = 3 cm^3/s$

$x = 2$cm, $\frac{dx}{dt} = ?$

$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$

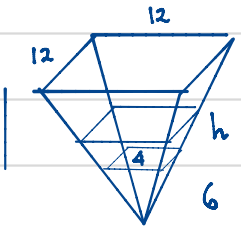
$\frac{dV}{dx} = 12\pi x - \pi x^2 = 24\pi - 4\pi$

$= 20\pi$

$3 \frac{cm^3}{dt} = 20\pi \times \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{3}{20\pi}$

21.



$\frac{6+h}{18} = \frac{n}{12}$

$n = \frac{(6+h)12}{18} = \frac{2}{3}(6+h)$

$\frac{6}{18} = \frac{x}{12}$

$x = \frac{6 \times 12}{18} = 4$ cm

$V_{bot} = \frac{1}{3} \times 4^2 \times 6 = 32 \ cm^3$

$V_{oil} = \frac{1}{3} \times \left[\frac{2}{3}(6+h)\right]^2 \times (6+h) - 32$

$= \frac{4}{27}(6+h)^3 - 32$

$\frac{dV}{dh} = \frac{4}{27} \times 3 (6+h)^2$

$\frac{dV}{dt} = 4.5 \ m^3/s$, $h = 8$m

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$4.5 = \frac{4}{9}(6+h)^2 \times \frac{dh}{dt}$

$4.5 = \frac{4}{9}(14)^2 \times \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{4.5 \times 9}{4 \times 14^2} = 0.0517 \approx 0.052$ m/s