Exercise 15 B

1a. $f(x) = 3x - x^3$

$f'(x) = 3 - 3x^2$

$f'(x) = 0$

$3 - 3x^2 = 0$

$3(1 - x^2) = 0$

$3(1+x)(1-x) = 0$

$x = -1, 1$

$f(-1) = -3 + 1 = -2 \quad (-1, -2)$

$f(1) = 3 - 1 = 2 \quad (1, 2)$

$f''(x) = -6x$

$f''(-1) = 6 \quad (-1,-2)$ is Min Point

$f''(1) = -6 \quad (1,2)$ is Max Point

1b. $f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x$

$f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0, x = 2$

$f(0) = 0 \quad (0,0)$

$f(2) = -4 \quad (2,-4)$

$f''(x) = 6x - 6$

$f''(0) = -6 \quad (0,0)$ is Max Point

$f''(2) = 6 \quad (2,-4)$ is Min Point

1c. $f(x) = 3x^4 + 1$

$f'(x) = 12x^3$

$f'(x) = 0$

$12x^3 = 0$

$x = 0$

$f(0) = 1 \quad (0,1)$

$f''(x) = 36x^2$

$f''(0) = 0 \rightarrow$ fail

$\underset{0}{-\quad +}$ $\quad (0,1)$ is Min Point

1.e. $f(x) = \frac{2}{x^4} - \frac{1}{x} = 2x^{-4} - x^{-1}$

$f''(x) = -8x^{-5} + x^{-2} = x^{-5}(x^3 - 8)$

$x^{-5}(x^3 - 8) = 0$

$x = 2, \ x \neq 0$

$f(2) = \frac{2}{16} - \frac{1}{2} = \frac{1}{8} - \frac{4}{3} = -\frac{3}{8} \quad (2, -\frac{3}{8})$

$f''(x) = 40x^{-6} - 2x^{-3}$

$f''(2) = \frac{40}{2^6} - \frac{2}{8} = \frac{40}{64} - \frac{1}{4} = \oplus \quad (2, -\frac{3}{8})$ Min Point

3d. $f(x) = 2x^3 - 3x^2 - 12x + 4$

$f'(x) = 6x^2 - 6x - 12$

$f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$6(x-2)(x+1) = 0$

$x = -1, 2$

$f(-1) = 11$

$f(2) = -16$

$f''(x) = 12x - 6$

$f''(-1) = -18 \quad (-1, 11)$ is Max

$f''(2) = 18 \quad (2, -16)$ is Min

f. $f(x) = x^2 + x^{-2}$

$f'(x) = 2x - 2x^{-3}$

$2x - 2x^{-3} = 0$

$2x^{-3}(x^4 - 1) = 0$

$2x^{-3}(x^2+1)(x+1)(x-1) = 0$

$x \neq 0, \ x = -1, x = 1$

$f(-1) = 2 \quad (-1, 2)$

$f(1) = 2 \quad (1, 2)$

$f''(1) = 8 \quad (1,2)$ Min point

$f''(-1) = 8 \quad (-1,2)$ Min point

g. $f(x) = \frac{1}{x} - \frac{1}{x^2} = x^{-1} - x^{-2}$

$f'(x) = -x^{-2} + 2x^{-3}$

$-x^{-2} + 2x^{-3} = 0$

$x^{-3}(2 - x) = 0$

$x = 2, \ f(2) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \quad (2, \frac{1}{4})$

$f''(x) = 2x^{-3} - 6x^{-4}$

$f''(2) = \frac{2}{8} - \frac{6}{16} = -\frac{2}{16} \rightarrow (2, \frac{1}{4})$ is Max Point

h. $f(x) = 2x^3 - 12x^2 + 24x + 6$

$f'(x) = 6x^2 - 24x + 24$

$6x^2 - 24x + 24 = 0$

$6(x^2 - 4x + 4) = 0$

$6(x-2)^2 = 0$

$x = 2, \ f(2) = 16 - 48 + 48 + 6 = 22 \quad (2, 22)$ Inflexion

$f''(x) = 12x - 24$

$f''(2) = 24 - 24 = 0$

$\underset{2}{+\quad +}$

2a. $y = 3x^4 - 4x^3 - 12x^2 - 3$

$f'(x) = 12x^3 - 12x^2 - 24x$

$12x^3 - 12x^2 - 24x = 0$

$12x(x^2 - x - 2) = 0$

$12x(x-2)(x+1) = 0$

$x = 0, -1, 2$

$f(0) = -3$ $(0,-3)$

$f(-1) = 3 + 4 - 12 - 3 = -8$ $(-1,-8)$

$f(2) = 48 - 32 - 48 - 3 = -35$ $(2,-35)$

$f''(x) = 36x^2 - 24x - 24$

$f''(-1) = 36 + 24 - 24$ $(-1,-8)$ Min

$f''(0) = -24$ $(0,-3)$ Max

$f''(2) = 72$ $(2,-35)$ Min

b. $y = x^3 - 3x^2 + 3x + 5$

$f'(x) = 3x^2 - 6x + 3$

$3x^2 - 6x + 3 = 0$

$3(x^2 - 2x + 1) = 0$

$3(x-1)^2 = 0$

$x = 1,\ f(1) = 6$ $(1,6)$

$f''(x) = 6x - 6$

$f''(1) = 0$ $(1,6)$ is $\oplus$ inflexion

$+ \quad \underset{1}{\circ} \quad +$

c. $y = 16x - 3x^3$

$f'(x) = 16 - 9x^2$

$16 - 9x^2 = 0$

$(4 + 3x)(4 - 3x) = 0$

$x = -\frac{4}{3}, \frac{4}{3}$

$f(-\frac{4}{3}) = -\frac{128}{9}$ $(-\frac{4}{3}, -\frac{128}{9})$

$f(\frac{4}{3}) = \frac{128}{9}$ $(\frac{4}{3}, \frac{128}{9})$

$f''(x) = -18x$

$f''(-\frac{4}{3}) = +$ $(-\frac{4}{3}, -\frac{128}{9})$ Min

$f''(\frac{4}{3}) = -$ $(\frac{4}{3}, \frac{128}{9})$ Max

d. $y = \frac{4}{x^2} - x = 4x^{-2} - x$

$f'(x) = -8x^{-3} - 1$

$-8x^{-3} - 1 = 0$

$8x^{-3} = -1$

$\frac{1}{x^3} = -\frac{1}{8}$

$x = -2$ $f(-2) = 3$

$f''(x) = 24x^{-4}$

$f''(-2) = +$ $(-2,3)$ Min

e. $y = \frac{4}{x} + x = 4.x^{-1} + x$

$f'(x) = -4x^{-2} + 1$

$1 - 4.x^{-2} = 0$

$x^{-2}(x^2 - 4) = 0$

$x^{-2}(x+2)(x-2) = 0$

$x = -2,\ f(-2) = -4$

$x = 2,\ f(2) = 4$

$f''(x) = 8x^{-3}$

$f''(-2) = \ominus$ $(-2,-4)$ Max

$f''(2) = \oplus$ $(2,4)$ Min

f. $y = \frac{1}{x} - \frac{3}{x^2} = x^{-1} - 3x^{-2}$

$f'(x) = -x^{-2} + 6x^{-3}$

$6x^{-3} - x^{-2} = 0$

$x^{-3}(6 - x) = 0$

$x = 6,\ f(6) = \frac{1}{6} - \frac{3}{36} = \frac{1}{12}$

$f''(x) = 2x^{-3} - 18x^{-4}$

$f''(6) = \frac{2}{216} - \frac{18}{1296} = \ominus$

$(6, \frac{1}{12})$ is Max point

g. $y = 2x^5 - 7$

$f'(x) = 10x^4$

$10x^4 = 0$

$x = 0,\ f(x) = -7$

$f''(x) = 40x^3$

$f''(0) = 0$

$+ \quad \underset{0}{\circ} \quad +$

$(0,-7)$ Inflexion

h. $y = 3x^4 - 8x^3 + 6x^2 + 1$

$f'(x) = 12x^3 - 24x^2 + 12x$

$12x^3 - 24x^2 + 12x = 0$

$12x(x^2 - 2x + 1) = 0$

$12x(x-1)^2 = 0$

$x = 0,\ x = 1$

$f(0) = 1$

$f(1) = 2$

$f''(x) = 36x^2 - 48x + 12$

$f''(0) = 12$ $(0,1)$ Min

$f''(1) = 0$

$(1,2)$ $\oplus$inflexion

$- \quad \underset{0}{\circ} \quad + \quad \underset{1}{\circ} \quad +$