EXERCISE 7B

1 d

$f(x) = 3x^2 - 5x + 7$

$f'(x) = 6x - 5 > 0$

$6x > 5$

$x > \frac{5}{6}$

e.

$f(x) = 5x^2 + 3x - 2$

$f'(x) = 10x + 3 > 0$

$10x > -3$

$x > -\frac{3}{10}$

f.

$f(x) = 7 - 4x - 3x^2$

$f'(x) = -4 - 6x > 0$

$-4 > 6x$

$x < -\frac{4}{6}$

$x < -\frac{2}{3}$

2d.

$f(x) = 2x^2 - 8x + 7$

$f'(x) = 4x - 8 < 0$

$4x < 8$

$x < 2$

e.

$f(x) = 4 + 7x - 2x^2$

$f'(x) = 7 - 4x < 0$

$7 < 4x$

$x > \frac{7}{4}$

f.

$f(x) = 3 - 5x - 7x^2$

$f'(x) = -5 - 14x < 0$

$-5 < 14x$

$x > -\frac{5}{14}$

3b.

$f(x) = 2x^3 - 18x + 5$

$f'(x) = 6x^2 - 18 > 0$

$6(x^2 - 3) > 0$

$x = \pm\sqrt{3}$

$+ \quad - \quad +$

$-\sqrt{3} \quad \sqrt{3}$

Increase : $x < -\sqrt{3}$ or $x > \sqrt{3}$

d.

$f(x) = x^3 - 3x^2 + 3x + 4$

$f'(x) = 3x^2 - 6x + 3 > 0$

$x^2 - 2x + 1 > 0$

$(x-1)^2 > 0$

ZV: $x = 1$

$x \neq 1$

f.

$f(x) = x^4 + 4x^3$

$f'(x) = 4x^3 + 12x^2 > 0$

$4x^2(x+3) > 0$

$x = 0, -3$

$- \quad + \quad +$

$-3 \quad 0$

$x > -3, x \neq 0$

3h.

$f(x) = 2x^5 - 5x^4 + 10$

$f'(x) = 10x^4 - 20x^3$

Stationary point : $f'(x) = 0$

$10x^4 - 20x^3 = 0$

$10x^3(x - 2) = 0$

$x = 2, x = 0$

$+ \quad - \quad +$

$0 \quad 2$

Increase : $x < 0$ or $x > 2$

4a. $f(x) = x^3 - 27x \ , \ x \geq 0$

$f'(x) = 3x^2 - 27$

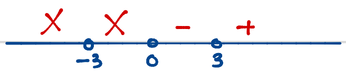
Stationary point : $f'(x) = 0$

$3x^2 - 27 = 0$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x = -3, 3 \ , \ x \geq 0$ (given)



$\therefore$ Decrease : $0 \leq x < 3$

c. $f(x) = x^3 - 3x^2 + 3x - 1$

$f'(x) = 3x^2 - 6x + 3$

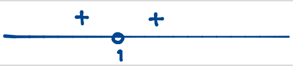
Stationary point : $f'(x) = 0$

$3x^2 - 6x + 3 = 0$

$3(x^2 - 2x + 1) = 0$

$3(x-1)^2 = 0$

$x = 1$



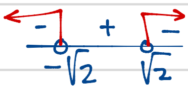
$\therefore$ Decrease : $x \in \phi$ (none)

d. $f(x) = 12x - 2x^3$

$f'(x) = 12 - 6x^2 < 0$

$6(2 - x^2) < 0$

$x = \pm\sqrt{2}$

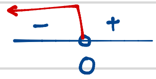


4b. $f(x) = x^4 + 4x^2 - 5$

$f'(x) = 4x^3 + 8x < 0$ for $x \geq 0$ (given)

$4x(x^2 + 2) < 0$

ZV: $x = 0$



Decrease is Not possible if $x \geq 0$

e. $f(x) = 2x^3 + 3x^2 - 36x - 7$

$f'(x) = 6x^2 + 6x - 36$

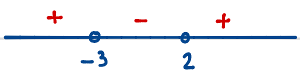
Stationary point : $f'(x) = 0$

$6x^2 + 6x - 36 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3, 2$



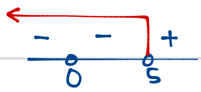
$\therefore$ Decrease : $-3 < x < 2$

f. $f(x) = 3x^4 - 20x^3 + 12$

$f'(x) = 12x^3 - 60x^2 < 0$

$12x^2(x - 5) < 0$

ZV: $x = 0, \ x = 5$



Decrease :

$x < 5, \ x \neq 0$

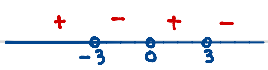
g. $f(x) = 36x^2 - 2x^4$

$f'(x) = 72x - 8x^3 = 8x(9 - x^2)$

$f'(x) = 0$

$8x(3+x)(3-x) = 0$

$x = 0, -3, 3$



$\therefore$ Decrease : $-3 < x < 0$,

$x > 3$

5b. $f(x) = x^{\frac{3}{4}} - 2x^{\frac{7}{4}}, x > 0$

$f'(x) = \frac{3}{4}x^{-\frac{1}{4}} - \frac{7}{2}x^{\frac{3}{4}}$

$= \frac{1}{4}x^{-\frac{1}{4}}(3 - 14x) = \frac{3-14x}{4\sqrt[4]{x}}$

ZV: $x = \frac{3}{14}$

$+ \quad \frac{3}{14} \quad -$

Decrease : $x > \frac{3}{14}$

Increase : $0 < x < \frac{3}{14}$

5f. $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}, x > 0$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{1}{2}x^{-\frac{3}{2}}(x-1)$

$= \frac{x-1}{2\sqrt{x^3}}$

ZV : $x = 1$

$- \quad 1 \quad +$

Decrease : $0 < x < 1$

Increase : $x > 1$

5d. $f(x) = x^{\frac{3}{5}}(x^2 - 13) = x^{\frac{13}{5}} - 13x^{\frac{3}{5}}$

$f'(x) = \frac{13}{5}x^{\frac{8}{5}} - \frac{39}{5}x^{-\frac{2}{5}}$

Stationary points: $f'(x) = 0$

$\frac{13}{5}x^{\frac{8}{5}} - \frac{39}{5}x^{-\frac{2}{5}} = 0$

$\frac{13}{5}x^{-\frac{2}{5}}(x^2 - 3) = 0$

$= \frac{13(x+\sqrt{3})(x-\sqrt{3})}{5\sqrt[5]{x^2}}$

ZV: $x = \pm\sqrt{3}$, $x = 0$ undefine

$+ \quad -\sqrt{3} \quad - \quad \sqrt{3} \quad +$

Decrease $-\sqrt{3} < x < \sqrt{3}$

Increase $x < -\sqrt{3}$, $x > \sqrt{3}$

6 b. $f(x) = 3x^2 + 12x + 5$

$f'(x) = 6x + 12$

ZV: $6x + 12 = 0$

$x = -2$, $y = 12 - 24 + 5 = -7$

$\begin{array}{cc} - & + \end{array}$ at -2

i $(-2, -7)$

ii Min point

iii $3x^2 + 12x + 5$

$3(x^2 + 4x) + 5$

$3[(x+2)^2 - 4] + 5$

$3(x+2)^2 - 7$ $(-2, -7)$

iii $y \geqslant -7$

d. $f(x) = 4 - 6x - x^2$

$f'(x) = -6 - 2x$

ZV: $-6 = 2x$

$x = -3$, $y = 4 + 18 - 9 = 13$

$\begin{array}{cc} + & - \end{array}$ at -3

i $(-3, 13)$

ii Max point

iii $-x^2 - 6x + 4 = -(x^2 + 6x - 4)$

$= -[(x+3)^2 - 9 - 4]$

$= -(x+3)^2 + 13$ $(-3, 13)$

iv. $y \leqslant 13$

f. $y = 1 - 4x - 4x^2 = -4x^2 - 4x + 1$

$y' = -4 - 8x = 0$

$-4 = 8x$

$x = -\frac{1}{2}$

$y = 1 + 2 - 4\left(\frac{1}{4}\right) = 2$

$-4(x^2 + x) + 1$

$-4\left[\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}\right] + 1$

$-4\left(x + \frac{1}{2}\right)^2 + 2$

$\left(-\frac{1}{2}, 2\right)$

ii Maximum point

iv $y \leqslant 2$

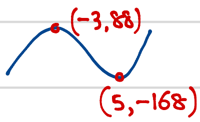
7. b $y = x^3 - 3x^2 - 45x + 7$

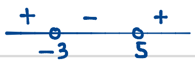
$y' = 3x^2 - 6x - 45 = 0$

$3(x^2 - 2x - 15) = 0$

$3(x-5)(x+3) = 0$

ZV: $x = 5, -3$





$x = 5, y = 125 - 75 - 225 + 7 = -168$ $(5, -168)$ Min Point

$x = -3, y = -27 - 27 + 135 + 7 = 88$ $(-3, 88)$ Max Point

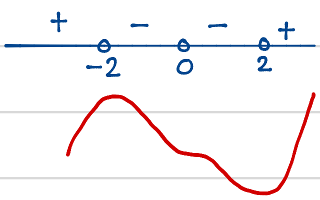
d. $y = 3x^5 - 20x^3 + 1$

$y' = 15x^4 - 60x^2 = 0$

$15x^2(x^2 - 4) = 0$

$15x^2(x+2)(x-2) = 0$

ZV: $x = 0, 2, -2$



$x = 0, y = 1$ $(0, 1)$ Inflexion

$x = -2, y = 65$ $(-2, 65)$ Max point

$x = 2, y = -63$ $(2, -63)$ Min point

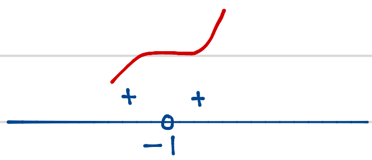
f. $y = x^3 + 3x^2 + 3x + 1$

$y' = 3x^2 + 6x + 3 = 0$

$3(x^2 + 2x + 1) = 0$

$3(x+1)^2 = 0$

ZV: $x = -1$



$x = -1, y = 0$ $(-1, 0)$ Inflection point

h. $f(x)=x^2+54x^{-1}$

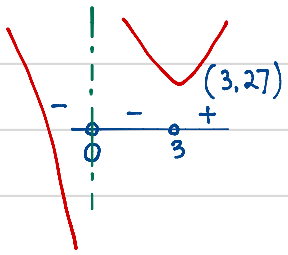
$f'(x)=2x-54x^{-2}=0$

$2x^{-2}(x^3-27)=0$

$\frac{2(x^3-27)}{x^2}=0$

$x\neq 0$

$x=3,\ y=9+\frac{54}{3}=9+18=27$



(3,27) Minimum point

j. $y=x-x^{\frac{1}{2}},\ x>0$

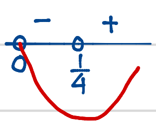
$y'=1-\frac{1}{2}x^{-\frac{1}{2}}$

$=x^{-\frac{1}{2}}\left(x^{\frac{1}{2}}-\frac{1}{2}\right)=0$

$\frac{\sqrt{x}-\frac{1}{2}}{\sqrt{x}}=0$

$\sqrt{x}=\frac{1}{2}$

$x=\frac{1}{4},\ 0$



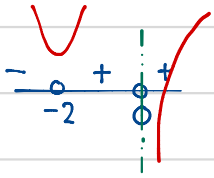
$x=\frac{1}{4},\ y=\frac{1}{4}-\frac{1}{2}=-\frac{1}{4}$

$\left(\frac{1}{4},-\frac{1}{4}\right)$ Min point

l. $y=x^2-\frac{16}{x}+5$

$y'=2x+\frac{16}{x^2}=2x^{-2}(x^3+8)=\frac{2(x^3+8)}{x^2}$

ZV : $x=-2,\ x\neq 0$



$x=-2,\ y=4+8+5=17$

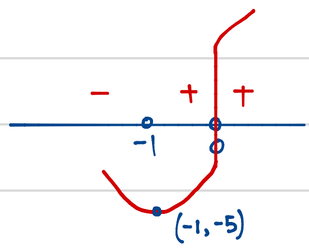
$(-2,17)$ Min Point

n. $y=x^{\frac{1}{5}}(x+6)=x^{\frac{6}{5}}+6x^{\frac{1}{5}}$

$y'=\frac{6}{5}x^{\frac{1}{5}}+\frac{6}{5}x^{-\frac{4}{5}}=\frac{6}{5}x^{-\frac{4}{5}}(x+1)$

$=\frac{6(x+1)}{5\sqrt[5]{x^4}}$

ZV : $x=-1,\ x=0$



$x=-1,\ y=\sqrt[5]{-1}(-1+6)=-1(5)=-5$

$(-1,-5)$ Min Point