Miscellaneous Exercise 15

1. $y = x^3 - 6x^2 + 9x + 6$

$y' = 3x^2 - 12x + 9 = 0$ $\quad y'' = 6x - 12$

$3(x^2 - 4x + 3) = 0$

$3(x-1)(x-3) = 0$

$x = 1,\ y = 10$ $\quad y'' = \ominus$ $\quad$ Max value = 10

$x = 3,\ y = 6$ $\quad y'' = \oplus$ $\quad$ Min value = 6

2. $f(x) = 16x + x^{-2}$

$f'(x) = 16 - 2x^{-3}$ $\quad f''(x) = 6x^{-4}$

$0 = 16 - 2x^{-3}$

$\frac{2}{x^3} = 16$

$x^3 = \frac{1}{8}$

$x = \frac{1}{2}$ $\quad f''(x) = \oplus$

$y = 8 + 4 = 12$

$(\frac{1}{2}, 12)$ Min point

Min value = 12

3. $f(x) = \sqrt{x} + \sqrt{30 - 5x} = x^{\frac{1}{2}} + (30-5x)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{5}{2}(30-5x)^{-\frac{1}{2}} = 0$

$\frac{1}{2}x^{-\frac{1}{2}} = \frac{5}{2}(30-5x)^{-\frac{1}{2}}$

$x^{-\frac{1}{2}} = 5(30-5x)^{-\frac{1}{2}}$

$\frac{1}{\sqrt{x}} = \frac{5}{\sqrt{30-5x}}$

$25x = 30 - 5x$

$30x = 30$

$x = 1$

Max Point

value: $1 + 5 = 6$

$f'' = -\frac{1}{4}x^{-\frac{3}{2}} + \frac{5}{4}(-5)(30-5x)^{-\frac{3}{2}}$

$= -\frac{1}{4}x^{-\frac{3}{2}} - \frac{25}{4}(30-5x)^{-\frac{3}{2}}$

$\frac{1}{x} = \frac{25}{30-5x}$

$f'' = -\frac{1}{4} - \frac{25}{4}(25)^{-\frac{3}{2}}$

$= -\frac{1}{4} - \frac{25}{4} \times \frac{1}{125} = -\frac{1}{4} - \frac{1}{20}$ $\ominus$

4. $y = \frac{1}{x} + \frac{1}{1-4x} = x^{-1} + (1-4x)^{-1}$

$f'(x) = -x^{-2} - (1-4x)^{-2}(-4)$

$= 4(1-4x)^{-2} - x^{-2}$

$f'(x) = 0$

$\frac{4}{(1-4x)^2} - \frac{1}{x^2} = 0$

$\frac{4x^2 - (1-4x)^2}{x^2(1-4x)^2} = 0$

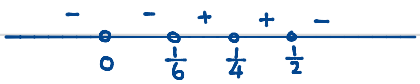
$\frac{4x^2 - (1 - 8x + 16x^2)}{x^2(1-4x)^2} = 0$

$\frac{-12x^2 + 8x - 1}{x^2(1-4x)^2} = 0$

$-\frac{(12x^2 - 8x + 1)}{x^2(1-4x)^2} = 0$

$-\frac{(6x-1)(2x-1)}{x^2(1-4x)^2} = 0$

$x = \frac{1}{6},\ x = \frac{1}{2}$ $\quad x \neq 0, x \neq \frac{1}{4}$



$f(\frac{1}{6}) = 6 + \frac{1}{1-\frac{4}{6}} = 6 + 3 = 9$ $\quad (\frac{1}{6}, 9)$ Min

$f(\frac{1}{2}) = 2 + \frac{1}{1-2} = 2 - 1 = 1$ $\quad (\frac{1}{2}, 1)$ Max

7. Perimeter = 10m

$2x + 2y + \pi x = 10\ m$

$2y = 10 - 2x - \pi x$

$y = 5 - x - \frac{\pi x}{2}$

$A = 2xy + \frac{\pi x^2}{2}$

$= 2x\left(5 - x - \frac{\pi x}{2}\right) + \frac{\pi x^2}{2}$

$= 10x - 2x^2 - \pi x^2 + \frac{\pi x^2}{2}$

$= 10x - 2x^2 - \frac{\pi x^2}{2} = 10x - \left(\frac{4+\pi}{2}\right)x^2$

$\frac{dA}{dx} = 10 - (4+\pi)x$

$\frac{dA}{dx} = 0$

$10 - (4+\pi)x = 0$

$10 = (4+\pi)x$

$x = \frac{10}{4+\pi}$ for Max Value of Area

$\frac{d^2A}{dx^2} = -(4+\pi) = \ominus$ value

$\therefore$ verified A is Max !

10. a. $y = x^4 - 8x^3 + 18x^2 + 4$

$y' = 4x^3 - 24x^2 + 36x$

$y'' = 12x^2 - 48x + 36 = 0$

$12(x^2 - 4x + 3) = 0$

$12(x-1)(x-3) = 0$

$x = 1,\ y = 15 \quad (1, 15)$

$x = 3,\ y = 31 \quad (3, 31)$

b. $y = x^2 - \frac{1}{x} + 2 = x^2 - x^{-1} + 2$

$y' = 2x + \frac{1}{x^2}$

$y'' = 2 - \frac{2}{x^3} = 0$

$2x^{-3}(x^3 - 1) = 0$

$x = 1,\ y = 2 \quad (1, 2)$

11. $y = 2x + \frac{1}{(x-1)^2}$

$y' = 2 - 2(x-1)^{-3} = 0$

$= \frac{2}{(x-1)^3}\left((x-1)^3 - 1\right) = 0$

$(x-1)^3 = 1$

$x - 1 = 1$

$x = 2$

$y'' = 6(x-1)^{-4}$

$x = 2,\ y'' = \oplus \rightarrow$ Min point