Miscellaneous Exercise 7

1. $z = 3x + 2y$ $\quad x, y, z$ positive values.

$xy = 600$

i $\quad y = \frac{600}{x}$

$z = 3x + 2\frac{600}{x} = 3x + \frac{1200}{x}$

ii $\quad \frac{dz}{dx} = 3 - \frac{1200}{x^2} = 0$

$\frac{3x^2 - 1200}{x^2} = 0$

$\frac{3(x^2 - 400)}{x^2} = 0$

$\frac{(x+20)(x-20)}{x^2} = 0$

ZV: $\quad x = -20, 20$

$+ \quad - \quad +$

$-20 \quad 20$

$x = -20,\ z = 3(-20) + \frac{1200}{-20} = -60 - 60 = -120$

$x = 20,\ z = 60 + 60 = 120$

$(-20, -120)$ — Max point Not eligible

$(20, 120)$ — Min point

6. $A = x^2 + y^2, \quad x + y = 10$

$y = 10 - x$

$A = x^2 + (10 - x)^2$

$= x^2 + 100 - 20x + x^2$

$= 2x^2 - 20x + 100$

$\frac{dA}{dx} = 4x - 20$

Stationary point $\frac{dA}{dx} = 0$

$4x - 20 = 0$

$4x = 20$

$x = 5$

Min Value = $\quad A = 5^2 + 5^2 = \underline{\underline{50}}$

9. $f(x) = (2x-5)^3 + x$

$f'(x) = 3(2x-5)^2 \times 2 + 1$

$= 6(2x-5)^2 + 1$

$f'(x)$ is always positive so

$f(x)$ is an increasing function

10. $y = x^3 + x^2$

$\frac{dy}{dx} = 3x^2 + 2x$

Stationary points: $\frac{dy}{dx} = 0$

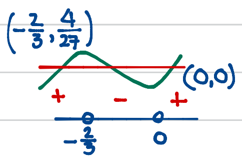
$3x^2 + 2x = 0$

$x(3x+2) = 0$

$x = 0, \quad x = -\frac{2}{3}, \quad y = \left(-\frac{2}{3}\right)^3 + \left(-\frac{2}{3}\right)^2$

$y = 0 \qquad = -\frac{8}{27} + \frac{4}{9}$

$(0,0) \qquad = \frac{4}{27} \quad \left(-\frac{2}{3}, \frac{4}{27}\right)$



$x^3 + x^2 = k$

$0 < k < \frac{4}{27}$

11. $y = 3x^4 - 4x^3 - 12x^2 + 10$

$\frac{dy}{dx} = 12x^3 - 12x^2 - 24x$

Stationary point: $\frac{dy}{dx} = 0$

$12x^3 - 12x^2 - 24x = 0$

$12x(x^2 - x - 2) = 0$

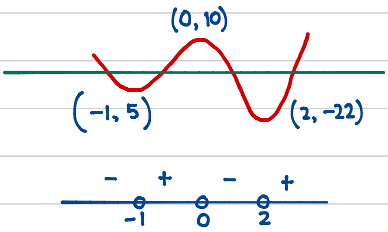
$12x(x-2)(x+1) = 0$

$x = 0, -1, 2$

$x = 0, y = 10$

$x = -1, y = 5$

$x = 2, y = -22$



$3x^4 - 4x^3 - 12x^2 + 10 = k$ has

a. exactly 4 roots if $5 < k < 10$

b. — 2 roots if $-22 < k < 5$

12. $6y + 8x = 48$

$6y = 48 - 8x$

i $y = 8 - \frac{4}{3}x$

ii $A = 4xy + 2xy$

$= 4x\left(8 - \frac{4}{3}x\right) + 2x\left(8 - \frac{4}{3}x\right)$

$= 32x - \frac{16}{3}x^2 + 16x - \frac{8}{3}x^2$

$= 48x - 8x^2$

iii $\frac{dA}{dx} = 48 - 16x = 0$

$48 = 16x$

$x = 3, \quad A = 48 \times 3 - 8 \times 9$

$= 144 - 72 = \underline{\underline{72}} \text{ m}^2$

13. i $2x + 2y + \frac{1}{4}\pi \cdot 2x = 60$

$2y = 60 - \frac{1}{2}\pi x - 2x$

$y = 30 - \frac{1}{4}\pi x - x$

ii $A = x \cdot y + \frac{1}{4}\pi x^2$

$= x\left(30 - \frac{1}{4}\pi x - x\right) + \frac{1}{4}\pi x^2$

$= 30x - \frac{1}{4}\pi x^2 - x^2 + \frac{1}{4}\pi x^2$

$= \underline{\underline{30x - x^2}}$ (Shown!)

iii $\frac{dA}{dx} = 30 - 2x = 0$

$x = 15$

iv $A = 30 \cdot 15 - 15^2 = 450 - 225 = 225 \text{ cm}^2 \rightarrow$ Max value

15. $80 = 4x + 2\pi r \Rightarrow r = \dfrac{80-4x}{2\pi} = \dfrac{40-2x}{\pi}$

(i) $A = x^2 + \pi r^2$

$= x^2 + \pi\left(\dfrac{40-2x}{\pi}\right)^2 = x^2 + \dfrac{1600 - 160x + 4x^2}{\pi}$

$= \dfrac{\pi x^2 + 1600 - 160x + 4x^2}{\pi} = \dfrac{(\pi+4)x^2 - 160x + 1600}{\pi}$ (shown!)

(ii) $\dfrac{dA}{dx} = 2\left(\dfrac{\pi+4}{\pi}\right)x - \dfrac{160}{\pi} = 0$

$2\left(\dfrac{\pi+4}{\pi}\right)x = \dfrac{160}{\pi}$

$x = \dfrac{160}{\pi} \times \dfrac{\pi}{2(\pi+4)}$

$= \dfrac{80}{\pi+4}$ //

16. Perimeter $= 2x + 2\pi r = 400$ m

$x = \dfrac{400 - 2\pi r}{2} = 200 - \pi r$

(i) $A = x(2r) + \pi r^2$

$= (200 - \pi r)(2r) + \pi r^2$

$= 400r - 2\pi r^2 + \pi r^2$

$= 400r - \pi r^2$ (shown!)

(ii) $\dfrac{dA}{dr} = 400 - 2\pi r$

$400 - 2\pi r = 0$

$400 = 2\pi r$

$r = \dfrac{400}{2\pi} = \dfrac{200}{\pi}$

If $r = \dfrac{200}{\pi}$, $x = 200 - \pi\left(\dfrac{200}{\pi}\right) = 0$

$+ \quad -$

$\dfrac{200}{\pi}$

$\therefore$ Max value.