Score :

Bukit Sion High School

Where the abundance in life flows……..

Subject : **Mathematics (Review)** **Class** : **11.1 / 11.2 / 11.3 / 11.4**
Day / date :____________, ______ **March 2020**
Topic : **Differential** **Name** : ________________________.

Instructions:
1. **Correction tape/fluid are not allowed.** 2. **Use black or dark blue ink.** 3. **Calculator is allowed.**

1. Differentiate the following with respect to x.
   a. $y = 3x^2 + 4x - 1$ [1]
   b. $f(x) = 4x + \frac{2}{x}$ [1]
   c. $y = \sqrt{x^3 - 2x}$ [1]
   d. $f(x) = \frac{6}{(3-2x)^3}$ [2]
   e. $y = \frac{2x^3+5}{2x+3}$ [2]

2. Calculate the gradient(s) of the curve at the given point.
   a. $y = 4x^2 - 6x + 1$, at $x = -2$ [2]
   b. $y = 2x^2 + 3x$, at $y = 2$ [4]

3. Find the coordinates of the point of the curve $y = x^3 - 3x^2 + 6x + 2$ at which the gradient is 3. [3]

4. Find the equation of the tangent to the curve $y = 3x^2 - 2x + 5$ which is perpendicular to the line $4y + x = 2$. [4]

5. A curve has equation $y = 2x^2 - 6x + 5$. Find the value of the constant k for which the line $y = 2x + k$ is a tangent to the curve. [4]

6. Given the curve $8y = x^2 - kx + 17$, calculate the value of k such that the tangents at the points with x-coordinates 5 and -3 respectively are perpendicular. [4]

7. Find the equation of the tangent at $x = -2$ to the curve with equation $y = 2x^2 - 3x + 2$. [4]

8. The normal to the curve $y = x^2$ has gradient $\frac{1}{2}$.

(i) Find the equation of the normal [3]

(ii) Find where the normal meets the curve. [3]

9. A metal cube is being expanded by heat. At the instant when the length of an edge is 2 cm, the volume of the cube is increasing at the rate of 0.012 cm^3 per second.
At what rate is the length of the edge increasing at this instant? [3]

10. Find the coordinates of the stationary points of the curve $y = x^4 - 8x^2 + 2$ and determine the nature of each point. [5]

11. An open rectangular box of height h cm has a horizontal rectangular base of sides x cm, and $2x$ cm. If the volume of the box is 36 cm^3,

(i) express h in terms of x [2]

(ii) show that the total surface area, A cm^2, of the box is given by

$$A = 2x^2 + \frac{108}{x}.$$ [1]

(iii) Find the value of x and h which make the total surface area a minimum. [3]

12. The point $P(3, 5)$ lies on the curve $y = \frac{1}{x-1} - \frac{9}{x-5}$.

(i) Find the x-coordinate of the point where the normal to the curve at P intersects the x-axis.
[5]

(ii) Find the x-coordinate of each of the stationary points on the curve and determine the nature of each stationary point. [6]

1a. $y = 3x^2 + 4x - 1$

$y' = 6x + 4$

b. $f(x) = 4x + \frac{2}{x}$

$f'(x) = 4 - \frac{2}{x^2}$

c. $y = \sqrt{x^3 - 2x} = (x^3 - 2x)^{\frac{1}{2}}$

$y' = \frac{1}{2}(x^3 - 2x)^{-\frac{1}{2}}(3x^2 - 2)$

d. $f(x) = \frac{6}{(3-2x)^3} = 6(3-2x)^{-3}$

$f'(x) = -18(3-2x)^{-4}(-2)$

$= 36(3-2x)^{-4}$

e. $y = \frac{2x^3+5}{2x+3}$

$y' = \frac{6x^2(2x+3) - 2(2x^3+5)}{(2x+3)^2}$

$= \frac{8x^3 + 18x^2 - 10}{(2x+3)^2}$ //

2. a. $y = 4x^2 - 6x + 1$

$m_t = y' = 8x - 6$

$x = -2,\ m_t = -16 - 6 = \underline{-22}$ [2]

b. $y = 2x^2 + 3x$

$m_t = f'(x) = 4x + 3$ [1]

$y = 2 = 2x^2 + 3x$ [1]

$2x^2 + 3x - 2 = 0$

$(2x-1)(x+2) = 0$

$x = \frac{1}{2},\ x = -2$

$m_t = 5,\ m_t = -5$ [2]

3. $y = x^3 - 3x^2 + 6x + 2$

$m_t = y' = 3x^2 - 6x + 6 = 3$ [1]

$3x^2 - 6x + 3 = 0$

$3(x-1)^2 = 0$

$x = 1,\ y = 6 \quad (1,6)$ [2]

4. $y = 3x^2 - 2x + 5$

$y' = m_t = 6x - 2 \perp 4y + x = 2,\ m = -\frac{1}{4}$ [1]

$m_t = 4 \qquad 4y = -x + 2$

$y = -\frac{1}{4}x + \frac{1}{2}$

$6x - 2 = 4$ [1]

$6x = 6$

$x = 1,\ y = 6$ [1]

$y - 6 = 4(x - 1)$

$\underline{\underline{y = 4x + 2}}$ [1]

5. $y = 2x^2 - 6x + 5$

$y = 2x + k$ is a tangent

$m_t = 2 = 4x - 6$ [1]

$4x = 8$

$x = 2,\ y = 1$ [2]

$1 = 2(2) + k$

$k = \underline{\underline{-3}}$ [1]

6. $8y = x^2 - kx + 17$

$y = \frac{1}{8}x^2 - \frac{k}{8}x + \frac{17}{8}$

$m_t = y' = \frac{1}{4}x - \frac{k}{8}$ [1]

$x = 5$ $m_{t1} = \frac{5}{4} - \frac{k}{8}$

$x = -3$ $m_{t2} = -\frac{3}{4} - \frac{k}{8}$

$m_{t1} \cdot m_{t2} = -1$

$\left(\frac{5}{4} - \frac{k}{8}\right)\left(-\frac{3}{4} - \frac{k}{8}\right) = -1$ [1]

$-\frac{15}{16} - \frac{5k}{32} + \frac{3k}{32} + \frac{k^2}{64} = -1$

$-\frac{15}{16} + 1 - \frac{2k}{32} + \frac{k^2}{64} = 0$

$\frac{k^2}{64} - \frac{4k}{64} + \frac{1}{16} = 0$

$k^2 - 4k + 4 = 0$

$(k-2)^2 = 0$

$\underline{\underline{k = 2}}$ [1]

7. $y = 2x^2 - 3x + 2$, $x = -2$

$y = 8 + 6 + 2 = 16$ $(-2, 16)$ [1]

$m_t = y' = 4x - 3$ [1]

$t = -2, y' = -11$ [1]

$y - 16 = -11(x+2)$

$y = -11x - 22 + 16$

$y = -11x - 6$ [1]

8. $y = x^2$, $m_n = \frac{1}{2}$

$m_t = 2x = -2$ [1]

$x = -1, y = 1$ [1]

$y - 1 = \frac{1}{2}(x+1)$

$y = \frac{1}{2}x + \frac{1}{2} + 1$ [1] oe

$\left.\begin{array}{l} y = \frac{1}{2}x + \frac{3}{2} \\ y = x^2 \end{array}\right\}$ $x^2 - \frac{1}{2}x - \frac{3}{2} = 0$ [1]

$2x^2 - x - 3 = 0$

$(2x-3)(x+1) = 0$

$x = -1, \frac{3}{2}$ $\left(\frac{3}{2}, \frac{9}{4}\right)$ [1]

$y = 1$, $y = \frac{9}{4}$ $(-1, 1)$ [1]

9. $V = s^3$, $\frac{dV}{dt} = 0.012\ cm^3/s$, $s = 2$ cm

$\frac{dV}{ds} = 3s^2 = 3 \cdot 2^2 = 12$ [1]

$\frac{dV}{dt} = \frac{dV}{ds} \times \frac{ds}{dt}$ [1]

$0.012 = 12 \times \frac{ds}{dt}$

$\frac{ds}{dt} = \frac{0.012}{12} = 0.001$ cm/s [1]

10. $y = x^4 - 8x^2 + 2$

$y' = 4x^3 - 16x = 0$ [1]

$4x(x^2 - 4) = 0$

$4x(x+2)(x-2) = 0$ [1]

$x = 0, -2, 2$

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$y'' = 12x^2 - 16$

$x = -2, y = 16 - 32 + 2 = -14$ $(-2, -14)$ Min Point

$x = 0, y = 2$ $(0, 2)$ Max Point

$x = 2, y = 16 - 32 + 2 = -14$ $(2, -14)$ Min Point

11 . (i) $V = x.2x.h = 36$ [1]

$2x^2h = 36$

$h = \frac{36}{2x^2} = \frac{18}{x^2}$ [1]

(ii) $A = 2x^2 + 6x.h = 2x^2 + 6x\left(\frac{18}{x^2}\right)$ [1]

$= 2x^2 + \frac{108}{x}$

(iii) $\frac{dA}{dx} = 4x - \frac{108}{x^2} = 0$ [1]

$4x^3 - 108 = 0$

$x^3 = 27$

$x = \sqrt[3]{27} = 3, \; h = 2$ [2]

12. $P(3,5)$ $y = (x-1)^{-1} - 9(x-5)^{-1}$

(i) $y' = m_t = -(x-1)^{-2} + 9(x-5)^{-2}$ [1]

$x = 3, \; m_t = -\frac{1}{4} + \frac{9}{4} = \frac{8}{4} = 2$ [1]

m_n at $P = -\frac{1}{2}$

$y - 5 = -\frac{1}{2}(x-3)$ [1]

x axis : $y = 0, \; -5 = -\frac{1}{2}(x-3)$ [1]

$10 = x - 3$

$x = 13$ $(13,0)$ [1]

(ii) $y' = -\frac{1}{(x-1)^2} + \frac{9}{(x-5)^2} = 0$ [1]

$\frac{9(x-1)^2 - (x-5)^2}{(x-1)^2(x-5)^2} = 0$

$\frac{[3(x-1)+(x-5)][3x-3-x+5]}{(x-1)^2(x-5)^2} = 0$

$\frac{(4x-8)(2x+2)}{(x-1)^2(x-5)^2} = 0$

$x = -1, \; x = 2$ [2]

$y'' = 2(x-1)^{-3} - 18(x-5)^{-3}$ [1]

$x = -1, \; y'' = -\frac{1}{4} + \frac{18}{6^3}$

$= -\frac{1}{4} + \frac{3}{36} = \ominus$ Max [1]

$x = 2, \; y'' = 2 - \frac{18}{27} = \oplus$ Min [1]